Non-classical logics
Summary of topics

1. **Classical logic**

   - **Propositional logic**
     - Syntax; semantics; models, validity, satisfiability, entailment, equivalence
     - Translation to CNF/DNF (in particular structure-preserving translations!)
     - Resolution; semantic tableaux
   - **First-order logic**
     - Syntax, semantics: models and assignments; validity, satisfiability; entailment and equivalence
     - Validity vs. unsatisfiability
     - Normal forms and Skolemization
     - General resolution (form of inference rules); unification (definition of a most general unifier; algorithm for computing a most general unifier) (No proofs required)
     - Semantic tableaux (No proofs required)

2. **Many-valued logics**

   - Propositional finitely valued logics: Syntax and semantics; models validity, satisfiability
   - First-order finitely valued logics: Syntax and semantics (interpretation of quantifiers!) models validity, satisfiability;
   - The many-valued logic $\mathcal{L}_3$ (you should know the truth tables of $\wedge, \vee, \sim, \neg$)
   - Functional completeness:
     - definition
     - criterion for functional completeness (criterion; idea of the proof)
     - examples of many-valued logics which are not functionally complete ($\mathcal{L}_3$, why?)
     - examples of functionally complete many-valued logics (classical logic, $\mathcal{L}_3^+$ (idea), Post logics (idea))

- Examples of many-valued logics (Post logics, Łukasiewics logics)
- Proof calculi
  - general proof calculi and inference systems (provability; soundness/completeness/refutational completeness: definitions)
  - proof calculus for $\mathcal{L}_3$ (you do not have to learn the inference rules by hard, but you should be able to use it for proving $\mathcal{L}_3$ theorems)
- Automated reasoning:
  - signed formulae; tableaux rules for finitely valued logics
  - translation to signed CNF; signed resolution: the two forms presented on the slides.
- Application in logic and verification (not required for the exam)

3. **Infinitely-valued logics**
   - Łukasiewicz logics (finitely valued; infinitely valued): Definitions; Main properties:
     - Link between tautologies of $\mathcal{L}_n$ and $\mathcal{L}_m$ if $(m-1)|(n-1)$ and the converse of this theorem (statement; idea of proof)
     - Relationship between the tautologies of $\mathcal{L}_{\aleph_0}$ and those of $\mathcal{L}_{\aleph_1}$
     - Relationship between the tautologies of $\mathcal{L}_{\aleph_0}$ and the tautologies of $\mathcal{L}_n$, $n \in \mathbb{N}$
   - "Fuzzy" logics: t-norms; definition; examples (Łukasiewicz logic, Gödel logic, Product logic)
   - Checking validity of formulae in fuzzy logics: reduction to checking constraints over $[0, 1]$ (Idea; applying the algorithm)

4. **Modal logic**
   - Motivation
   - Syntax
     - Signature; formulae
     - Proof theory (inference systems - general definitions; soundness, completeness)
     - Inference system for modal logics; proofs
   - The modal system K (you do not need to learn the axioms of the other systems of modal logic, but you should be able to use them for proving theorems in the modal logic K)
   - Semantics:
     - Kripke frames and Kripke structures
     - Models, validity, satisfiability
     - Entailment (global entailment; local entailment; the deduction theorem)

- Correspondence theory
  - Correspondence theorem: the general result linking the property $C(m, n, j, k)$ of R and axiom $\Diamond^m \Box^n P \rightarrow \Box^j \Diamond^k P$
  - Application: correspondence theorems for the properties/axiom schemata in the list given on the slides.
- First-order definability
  - Irreflexivity does not correspond to the validity of any modal schema (idea of proof)
- Theorem proving (for the modal logic K):
  - Axiom system (soundness; completeness: only the main idea of the proof – construction of canonical models)
  - semantic tableaux (soundness and completeness: only idea)
  - translation to first-order logic (+ resolution)
- Decidability of the modal logic K (idea of the proof – construction of filtrations; properties of filtrations; finite model property + computable bound on size of models; sketch of decision procedure; extensions to other modal logics)

5. **Description logics:** ALC (definition; link with the modal logic K); SHIQ (definition)

6. **Dynamic logic** (main definitions; syntax and semantics; completeness and finite model property: only the results and the very general idea of the proof, no details)