Universität Koblenz-Landau
FB 4 Informatik

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Exercises for

Advances in Theoretical Computer Science

Exercise Sheet 1

Due at 22.10.12, 09:00 s.t.

Exercise 1.1

Get acquainted with the following definitions of Turing Machines and related concepts:
A *Turing Machine* (TM) $\mathcal{M}$ is a tuple $\mathcal{M} = (K, \Sigma, \delta, s)$ with

- K a finite set of states, $h \notin K$,
- Σ an alphabet, $L, R \notin \Sigma$ and $\# \in \Sigma$,
- $\delta : K \times \Sigma \rightarrow (K \cup \{h\}) \times (\Sigma \cup \{L, R\})$ a transition function, and
- $s \in K$ an initial state.

The transition $\delta(q, a) = (q', x)$ describes that if a TM is in state $q \in K$ and the symbol $a \in \Sigma$ is read, the TM changes its state to $q' \in K \cup \{h\}$ and

- moves the head one step to the left, iff $x = L$
- moves the head one step to the right, iff $x = R$
- does not move the head but prints the symbol $b \in \Sigma$ on the tape, iff $x = b \in \Sigma$

A *configuration* C of a TM $\mathcal{M} = (K, \Sigma, \delta, s)$ is a string $C = q, w\underline{a}u$, with

- $q \in K \cup \{h\}$, the current state,
- $w \in \Sigma^*$, the tape contents left of the head,
- $a \in \Sigma$, the tape content under the head (the current symbol),
- $u \in \Sigma^*(\Sigma - \{\#\}) \cup \{\varepsilon\}$, the tape contents right of the head,

The *initial configuration* C_0 of $\mathcal{M}$ is defined as $C_0 = s, \#w\underline{\#}$ with *input* $w \in \Sigma^*$.
$C_2 = w_2a_2u_2$ is a *successor configuration* of $C_1 = w_1a_1u_1$, written as $C_1 \vdash_{\mathcal{M}} C_2$, iff there is a transition $\delta(q_1, a_1) = (q_2, b)$ and:

Case 1: $b \in \Sigma$**.** Then $w_1 = w_2, u_1 = u_2, a_2 = b$.

Case 2: $b = L$**.** Then for w_2 and $a_2 : w_1 = w_2a_2$. For u_2 : If $a_1 = \#$ and $u_1 = \varepsilon$, then $u_2 = \varepsilon$, otherwise $u_2 = a_1u_1$.

Case 3: $b = R$**.** Then for $w_2 = w_1a_1$. For a_2 and u_2 : If $u_1 = \varepsilon$, then $u_2 = \varepsilon$ and $a_2 = \#$, otherwise $u_1 = a_2u_2$.

$C_0 \vdash^*_{\mathcal{M}} C_n$ is called *computation*, iff for all C_i with $0 \leq i < n$, C_{i+1} is a successor configuration of C_i.

Exercise 1.2

a) Define a Turing Machine $\mathcal{M}_a$ that accepts all words $w \in \{|\}^*$ with an even length, i.e. $\mathcal{M}_a$ holds, iff w has even length, otherwise $\mathcal{M}_a$ does not terminate.

b) Define a Turing Machine $\mathcal{M}_d$ that decides if a word $w \in \{|\}^*$ has an even length.

   $s, \#w\underline{\#} \vdash^*_{\mathcal{M}_d} h, \#Y\underline{\#}$, iff w has even length,
   $s, \#w\underline{\#} \vdash^*_{\mathcal{M}_d} h, \#N\underline{\#}$, iff w has odd length.

c) Define a Turing Machine $\mathcal{M}_i$ that adds one $|$ to an input word $w \in \{|\}^*$.

   $s, \#w\underline{\#} \vdash^*_{\mathcal{M}_i} h, \#w|\underline{\#}$.

You can decide to give the formal definition of the Turing Machines or to draw it in the flow chart notation.

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The submission of the solutions is not compulsory. If you want to submit your solutions, please do so until 22.10.12, 09:00 s.t.. Joint solutions prepared by up to two persons are allowed. Please do not forget to write your name on your solution.
Submission possibilities:

- By e-mail to `mbender@uni-koblenz.de` with the keyword “Homework ACTCS” in the subject.
- Put it in the box in front of Room B 222 (if you prefer to submit the written exercise like this please tell me such that I can prepare such a box).