Universität Koblenz-Landau
FB 4 Informatik

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Exercises for

Advances in Theoretical Computer Science

Exercise Sheet 2

Due at 30.10.12, 09:00 s.t.

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Exercise 2.1

Define a 3-tape Turing Machine $\mathcal{M}_{mul}$ that calculates the product of the numbers n_1, n_2 in unary notation.

$$s, \#|^{n_1}\#|^{n_2}\underline{\#}, \underline{\#}, \underline{\#} \vdash^*_{\mathcal{M}_{mul}} h, \#|^{n_1 * n_2}\underline{\#}, \underline{\#}, \underline{\#}.$$

You can decide to give the formal definition of the Turing Machines or to draw it in the flow chart notation. You can introduce „auxiliary Turing Machines", to ease the task — as long as you define what they are doing.

Exercise 2.2

Define a Turing Machine (you are free to choose any variant you like) that accepts the language $L = \{|^n \mid n \text{ is prime}\}$.
You can decide to give the formal definition of the Turing Machines or to draw it in the flow chart notation. You can introduce „auxiliary Turing Machines", to ease the task — as long as you define what they are doing.

Exercise 2.3

Prove or refute the following statements:

a) The union of two recursively enumerable languages is recursively enumerable.

b) The intersection of two recursively enumerable languages is recursively enumerable.

c) The concatenation of two recursively enumerable languages is recursively enumerable.

d) The complement of a recursively enumerable language is recursively enumerable.

e) The union of two decidable languages is decidable.

f) The intersection of two decidable languages is decidable.

g) The concatenation of two decidable languages is decidable.

h) The complement of a decidable language is decidable.

Exercise 2.4

Let L_1, L_2, L_3 be languages, where L_2 is recursively enumerable and L_3 is decidable. Prove or refute the following statements:

1. If $L_1 \subseteq L_3$, then L_1 is decidable.
2. If $L_3 \subseteq L_1$, then L_1 is decidable.
3. If $L_1 \subseteq L_2$, then L_1 is recursively enumerable.
4. If $L_2 \subseteq L_1$, then L_1 is recursively enumerable.