

Exercises for
Advances in Theoretical Computer Science
Exercise Sheet 3

Due at 06.11.12, 09:00 s.t.

Exercise 3.1

Decide for each of the following problems if it is decidable or not. Justify your answer.

- a) $P_1 := \{n \in \mathbb{N} \mid \mathcal{M}_n \text{ does not halt on empty input}\}$
- b) $P_2 := \{n \in \mathbb{N} \mid L(\mathcal{M}_n) = \emptyset\}$
- c) $P_3 := \{n \in \mathbb{N} \mid L(\mathcal{M}_n) \text{ is a finite language}\}$
- d) $P_4 := \{(m, n) \in \mathbb{N} \times \mathbb{N} \mid L(\mathcal{M}_m) \cap L(\mathcal{M}_n) = \emptyset\}$
- e) $P_5 := \{(m, n) \in \mathbb{N} \times \mathbb{N} \mid L(\mathcal{M}_m) \subseteq L(\mathcal{M}_n)\}$
- f) $P_6 := \{(n, w) \in \mathbb{N} \times \Sigma^* \mid \text{For input } w, \mathcal{M}_n \text{ does not reach another configuration after } s, \#w\# \text{ where the head is on a blank } (\#)\}$
- g) $P_7 := \{(n, w, s) \in \mathbb{N} \times \Sigma^* \times \mathbb{N} \mid \mathcal{M}_n \text{ halts on input } w \text{ after at most } s \text{ steps}\}$

Remarks:

- \mathcal{M}_n denotes the Turing Machine with Gödel number n .
- $L(\mathcal{M})$ is the language accepted by the Turing Machine \mathcal{M} (i.e. the set of all words accepted by \mathcal{M}).
- You are allowed to use the undecidability of the Halting Problems.
- If you have proven the (un-)decidability of P_i , you may use this result for the other tasks.

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The submission of the solutions is not compulsory. If you want to submit your solutions, please do so until 06.11.12, 09:00 s.t.. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to mbender@uni-koblenz.de with the keyword "Homework ACTCS" in the subject.
- Put it in the box in front of Room B 222.