Universität Koblenz-Landau

FB 4 Informatik

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Exercises for
"Advances in Theoretical Computer Science"
Exercise sheet 5
Due on 20.11.12, 09:00 s.t.

Exercise 5.1:
Let P be the following LOOP-program.

```
loop x1 do
   x3 := x3 + 1
end;
// (1)
loop x3 do
   x2 := x2 + x3
end;
//(2)
x3 := 0
```

(1) Fill in the following table with the values of the registers x_1, x_2, x_3 at points (1) and (2) in the program: (i) for input 3; (ii) for input 5.

Input 3	x_1	x_2	x_3
(1)			
(2)			

Input 5	x_1	x_2	x_3
(1)			
(2)			

(2) Which is the output of P for input 3? Which is the output of P for input 5?

(3) Which function $f : \mathbb{N} \to \mathbb{N}$ is computed by P?

Exercise 5.2:

(1) Let $f : \mathbb{N} \to \mathbb{N}$ be a bijective function which is WHILE computable. Show that its inverse, $f^{-1} : \mathbb{N} \to \mathbb{N}$, is WHILE computable as well.
Can we find any GOTO computable bijection $g : \mathbb{N} \to \mathbb{N}$ for which $g^{-1} : \mathbb{N} \to \mathbb{N}$ is not GOTO computable?

(2) Let $f : \mathbb{N} \times \mathbb{N}$ be a bijective function which is WHILE computable. Let P be the WHILE program which computes f. Write a WHILE program, which uses P, with the property that started with input n_1 in register x_1 it ends with value n_2 in register x_2 and value n_3 in register x_3, where n_2 and n_3 are such that $f(n_2, n_3) = n_1$.

Remark: You are allowed to use all instructions introduced in the lecture (defined as LOOP programs, hence expressible also as WHILE programs).

Exercise 5.3:
Write a GOTO program which computes the function $q : \mathbb{N} \to \mathbb{N}$ defined for every $n \in \mathbb{N}$ by:

$$q(n) \text{ is the sum of the digits in } n.$$

Remark: You are allowed to use all instructions introduced in the lecture (defined as LOOP programs, hence expressible also as WHILE programs and as GOTO programs).

Exercise 5.4:
Let P be the following GOTO program:

```
1: x4 := x1;

2: if x4 = 0 goto 10;
3: x5 := x2;

4: if x5 = 0 goto 8;
5: x3 := x3 + 1;
6: x5 := x5 - 1;
7: if x6 = 0 goto 4;

8: x4 := x4 - 1;
9: if x6 = 0 goto 2;

10: x5 := x5 - 1
```

(1) Which function $f : \mathbb{N}^2 \to \mathbb{N}$ is computed by P?
(2) Give an equivalent WHILE program.

Remark: You are allowed to use all instructions introduced in the lecture (defined as LOOP programs, hence expressible also as WHILE programs).

Exercise 5.5:
The proof of the fact that $\text{WHILE}^{\text{part}} \subseteq \text{GOTO}^{\text{part}}$ given in the lecture from 15.11.2012 (page 16) can be used to show that

- if P is while $x_i \neq 0$ do P_1 end and
- if P_1' is a GOTO program which simulates P_1

then the following GOTO program simulates P:

j_1 : if $x_i = 0$ goto j_3;
P_1';
j_2 : if $x_n = 0$ goto j_1; // Since $x_n = 0$ unconditional jump
j_3 : $x_n := x_n - 1$

where x_n is a new register, and j_1, j_2, j_3 are new indices (do not occur in P_1').

Prove, with the help of this result (by induction on the structure of WHILE programs) that for every WHILE-program there exists a GOTO-program with the same semantics.

The submission of the solutions is not compulsory. If you want to submit your solutions, please do so until 20.11.12, 09:00 s.t.. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.
Submission possibilities:

- By e-mail to mbender@uni-koblenz.de with the keyword “Homework ACTCS” in the subject.
- Put it in the box in front of Room B 222.