Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans
Dipl. Inf. Markus Bender

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Exercises for
"Advances in Theoretical Computer Science"
Exercise sheet 10
Due on 8.1.13, 09:00 s.t.

Exercise 10.1:
Let $f : \mathbb{N} \to \mathbb{N}$ be the function defined by:

$$f(n) = \begin{cases} m & \text{if } n = 3^m, \quad m \in \mathbb{N} \\ \text{undefined} & \text{otherwise} \end{cases}$$

(1) Prove that f is μ-recursive.

(2) Is f primitive recursive?

Exercise 10.2:
Let $h : \mathbb{N} \to \mathbb{N}$ be a primitive recursive function, and let $k \geq 2$ be a natural number.

Let $f : \mathbb{N} \to \mathbb{N}$ be defined by:

$$\begin{array}{rcll} f(i) & = & h(i) & \text{for all } i \leq k \\ f(i) & = & f(i-k) * f(i-k-1) & \text{for all } i > k \end{array}$$

Show that f is primitive recursive.

Exercise 10.3:
Let $k \geq 0$ be a natural number. Let $g_0, g_1, \dots, g_k : \mathbb{N}^p \to \mathbb{N}$, and $h : \mathbb{N}^{p+2} \to \mathbb{N}$ be primitive recursive functions.

Let $f : \mathbb{N}^{p+1} \to \mathbb{N}$ be defined by:

$$\begin{array}{rcll} f(\mathbf{n}, m) & = & g_m(\mathbf{n}) & \text{for all } m \leq k \\ f(\mathbf{n}, m+1) & = & h(\mathbf{n}, m, f(\mathbf{n}, m-k)) & \text{for all } m \geq k \end{array}$$

Show that f is primitive recursive.

Exercise 10.4:
A k-ary predicate over $\mathbb{N}$ can be also regarded as a subset of $\mathbb{N}^k$: $P \subseteq \mathbb{N}^k$. We write $P(n_1, \dots, n_k)$ if $(n_1, \dots, n_k) \in P$.

The characteristic function of P is the map $\chi_P : \mathbb{N}^k \rightarrow \{0, 1\}$ with:

$$\chi_P(n_1, \ldots, n_k) = \begin{cases} 1 & \text{if } P(n_1, \ldots, n_k) \\ 0 & \text{otherwise} \end{cases}$$

The predicate P is primitive recursive iff its characteristic function χ_P is primitive recursive.

Prove:

(1) If P, R are k-ary primitive recursive predicates then also the predicates P_1, P_2, P_3 and P_4 are primitive recursive, where:

- $P_1(n_1, \ldots, n_k)$ iff $(P(n_1, \ldots, n_k)$ and $R(n_1, \ldots, n_k))$
- $P_2(n_1, \ldots, n_k)$ iff $(P(n_1, \ldots, n_k)$ or $R(n_1, \ldots, n_k))$
- $P_3(n_1, \ldots, n_k)$ iff $(\exists i \leq n_k$ s.t. $P(n_1, \ldots, n_{k-1}, i))$
- $P_4(n_1, \ldots, n_k)$ iff $(\forall i \leq n_k \;\; P(n_1, \ldots, n_{k-1}, i))$

(2) The predicates B, S are primitive recursive, where:

- $B(n_1, n_2, n_3)$ iff (n_2 is between n_1 and n_3 w.r.t. the usual order on $\mathbb{N}$)
- $B(n_1, n_2, n_3)$ iff (n_3 is the sum of the squares of n_1 and n_2)

Exercise 10.5:
Let P be a GOTO program which computes the function $f : \mathbb{N} \rightarrow \mathbb{N}$.

Let $g : \mathbb{N} \rightarrow \mathbb{N}$ be such that $g(n)$ is the number of instructions P needs to compute $f(n)$.

Prove that if g is LOOP computable then f is LOOP computable as well.

The submission of the solutions is not compulsory. If you want to submit your solutions, please do so until 8.1.13, 09:00 s.t.. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.
Submission possibilities:

- By e-mail to mbender@uni-koblenz.de with the keyword “Homework ACTCS” in the subject.
- Put it in the box in front of Room B 222.