Universität Koblenz-Landau

FB 4 Informatik

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January 17, 2013

Exercises for
"Advances in Theoretical Computer Science"
Exercise sheet 12
Due on 22.1.13, 09:00 s.t.

Exercise 12.1:
Consider the correspondence system $P = \{\underbrace{(a, baa)}_{(p_1,q_1)}, \underbrace{(ab, aa)}_{(p_2,q_2)}, \underbrace{(bba, bb)}_{(p_3,q_3)}\}$.

- Does P have a solution with start 1? Does P have a solution with start 2?
- Find a solution for P.

Exercise 12.2:
Let $\Sigma = \{a, b\}$ and let $R = \{(a \to baa), (ab \to aa), (bba \to bb)\}$.

(1) Let $G_1 = (\Sigma, R)$ be a semi-Thue system. Is it true that $baa \Rightarrow^*_{G_1} bb$?

(2) Let $G_2 = (\Sigma, R)$ be a Post normal system. Is it true that $baa \Rightarrow^*_{G_2} bb$?

If a computation exists write all the steps, indicating the numbers of the rules in R used and underlining the occurrence of the left hand side of the rule in the current word.

Consider now the semi-Thue system G_1 and the words $w' = baa$ and $w'' = bb$.

(3) Construct the correspondence system $P_{G_1,w',w''}$ as explained on Slide 19 of the lecture from 17.01.2013. Assume that rule 4 is $(X, Xw'X)$.

(4) Construct a solution for $P_{G_1,w',w''}$ with start 4 using the derivation $baa \Rightarrow^*_{G_1} bb$.

Hint: In (4) use the idea presented in the Example on pages 20-26 of the slides from 17.01.2013 (cf. also pages 313-315 in the book "Theoretische Informatik (Auflage 3)" by Erk and Priese).

Exercise 12.3:
Assume that Σ consists of one element only. Show that in this case the Post correspondence problem with alphabet Σ is decidable.

Exercise 12.4:
Prove that for every alphabet Σ with $|\Sigma| \geq 2$ it is undecidable whether for DCFL languages L_1, L_2 we have $L_1 \subseteq L_2$.

Hint: Reduction to the problem of testing emptiness for intersection of DFCL languages. Use the fact that the complement of a DCFL language is a DCFL language.

Exercise 12.5:
A map $h : \Sigma_1^* \to \Sigma_2^*$ is a monoid homomorphism if it has the property that for all words $w, w' \in \Sigma_1^*$, $h(w_1 w_2) = h(w_1)h(w_2)$.

Prove that the following problem is undecidable:

> Let $f, g : \Sigma_1^* \to \Sigma_2^*$ be monoid homomorphisms. Assume that $\Sigma_1 = \{a_1, \dots, a_n\}$.
>
> Is there a word $w \in \Sigma_1^*$ such that $f(w) = g(w)$?

Hint: Use the fact that the Post correspondence problem is undecidable. (Note that f and g are completely described by their values on $a_1, \dots, a_n$.)

The submission of the solutions is not compulsory. If you want to submit your solutions, please do so until 22.1.13, 09:00 s.t.. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.
Submission possibilities:

- By e-mail to mbender@uni-koblenz.de with the keyword "Homework ACTCS" in the subject.
- Put it in the box in front of Room B 222.