Universität Koblenz-Landau

FB 4 Informatik

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Exercises for
"Advances in Theoretical Computer Science"
Exercise sheet

Complexity

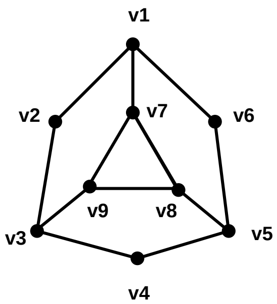
Exercise 1 (2+2+4+2 = 10p)

Let $G = (V, E)$ be an undirected graph. A set of vertices $V_1 \subseteq V$ is an independent set if there are no edges between any two of these vertices, i.e. if

$$\text{for all } v_1, v_2 \in V \text{ (if } v_1, v_2 \in V_1 \text{ then } (v_1, v_2) \notin E).$$

Let IND-SET be the language $\{(G, k) \mid G \text{ is a graph with an independent set of size } k\}$.

(1) Let G be the following graph. Does G have an independent set of size 3? Does G have an independent set of size 4? Does G have an independent set of size 5? In case your answer is positive give an example of an independent set (with 3, 4, or 5 elements).



Independent set of size 3 exists	yes ☐ no ☐	Example:
Independent set of size 4 exists	yes ☐ no ☐	Example:
Independent set of size 5 exists	yes ☐ no ☐	Example:

(2) Is IND-SET in NP? Justify your answer briefly (you do not need to construct a Turing machine).

(3) Let f be the map which associates with every pair (G, k), where $G = (V, E)$ is an undirected graph and $k \in \mathbb{N}$, the pair $f(G, k) = (G', k)$, where $G' = (V, E')$ is the complement of G, i.e. $(x, y) \in E'$ iff $(x, y) \notin E$.

Use f to prove that there is a polynomial reduction from Clique to IND-SET.

(4) Is IND-SET NP complete? Justify your answer.

Exercise 2

Consider the following problem: Given a set of courses, a list of conflicts between them, and a positive integer k; is there an exam schedule consisting of k dates such that there are no conflicts between courses which have examinations on the same date?

The language is

SCHEDULE = $\{(S, C, k)$ | S is a set of courses,
C is a set of conflicts between courses (a set of two-element subsets of S),
$k \in \mathbb{N}$ s.t. there exists an exam schedule consisting of k examinations dates
with no conflicts between courses which have examinations on the same day$\}$.

(1) Which of the tuples (S, C, k) below is an instance of SCHEDULE?

$(\ \{a, b, c, d\},\ \{\{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}\}, 2)$	☐
$(\ \{a, b, c, d\},\ \{\{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}\}, 3)$	☐

(2) Is SCHEDULE in NP? Justify your answer briefly (you do not need to construct a Turing machine).

(3) In the lecture we studied the k-colorability problem:

$$k\text{-colorability} = \{G \mid G \text{ undirected graph which is colorable with at most } k \text{ colors }\}.$$

It is known that for $k \geq 3$, k-colorability is NP complete.

Let f be the map which associates with every undirected graph $G = (V, E)$ (where the set of edges is regarded as a set of 2-element subsets of V) the tuple $(V, E, 3)$. Prove that f defines a polynomial reduction of 3-colorability to SCHEDULE.

(4) Is SCHEDULE NP complete? Justify your answer.