Universität Koblenz-Landau

* **FB 4 Informatik**

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Exercises for
"Advances in Theoretical Computer Science"
Exercise sheet *

1 Register machines

The following instructions are considered to be WHILE instructions in Exercises 1.1, 1.2, 2.1, 2.2:

$x_i := c$	$x_i := c$ op x_j	$P_1; P_2$	Here: x_i, x_j, x_k are registers
$x_i := x_j$	$x_i := x_j$ op c	while $x_i \neq 0$ do P_1 end	c is a constant
	$x_i := x_j$ op x_k	if $x_i = 0$ then P_1 end	op $\in \{+, -, *\}$
			and P_1, P_2 are WHILE programs.

The following instructions are considered to be GOTO instructions in Exercises 1.2 and 2.2:

$x_i := c$	$x_i := c$ op x_j	goto l	Here: x_i, x_j, x_k are registers
$x_i := x_j$	$x_i := x_j$ op c	if $x_i = 0$ goto l	c is a constant
	$x_i := x_j$ op x_k		op $\in \{+, -, *\}$
			and l is a label.

A GOTO program has the form $l_1 : B_1, \ldots, l_k : B_k \quad (k \geq 1)$
where $B_1, \ldots, B_k$ are GOTO instructions and $l_1, \ldots, l_k$ are labels.

Exercise 1.1 (4 + 4 = 8p)

Let $f : \mathbb{N} \times \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ be defined as follows:

$$f(n, m, k) = \begin{cases} \lfloor \sqrt[n]{k^m} \rfloor & \text{if } n \neq 0 \text{ and } (m \neq 0 \text{ or } k \neq 0) \\ \text{undefined} & \text{if } n = 0 \text{ or } (m = 0 \text{ and } k = 0) \end{cases}$$

(1) Write a pseudocode program for f which contains only the arithmetical operations $+, -, *$. The instructions used in this pseudocode program are not restricted to the ones above.

(2) Give a WHILE program which computes the function f.

Exercise 1.2 (2+4+4 = 10p)

Let P be the following WHILE program:

```
x4 := 10 − x1;
x5 := 1 − x4;
while x5 ≠ 0 do
        x5 := x5 + 1
end;
x4 := x2 − 1;
x5 := x1;
x3 := x1;
while x4 ≠ 0 do
        x5 := x5 ∗ 10
        x3 := x3 + x5
        x4 := x4 − 1
end;
x5 := 0
```

(1) Which value does P compute on input $x_1 = 2, x_2 = 3$? Which value does P compute on input $x_1 = 3, x_2 = 0$?

(2) Which function $f : \mathbb{N}^2 \to \mathbb{N}$ is computed by P?

(3) Use the transformation presented in the lecture to construct a GOTO program which has the same semantics as P.

Exercise 2.1 (4 + 4 = 8p)

Let $f : \mathbb{N} \times \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ be defined as follows:

$$f(n, m, k) = \begin{cases} (\lfloor \log_m(k) \rfloor)^n & \text{if } n > 1, m > 1 \text{ and } k > 0 \\ \text{undefined} & \text{otherwise} \end{cases}$$

(1) Write a pseudocode program for f which contains only the arithmetical operations $+, -, *$. The instructions used in this pseudocode program are not restricted to the ones above.

(2) Give a WHILE program which computes the function f.

Exercise 2.2 (2+4+4 = 10p)

Let P be the following GOTO program:

$$
\begin{aligned}
1: &\quad x_4 := 10 - x_1;\\
2: &\quad x_5 := 1 - x_4;\\
3: &\quad \text{if } x_5 = 0 \text{ goto } 6;\\
4: &\quad x_5 := x_5 + 1;\\
5: &\quad \text{goto } 3;\\
6: &\quad x_6 := x_2;\\
7: &\quad x_5 := x_4;\\
8: &\quad x_3 := x_1;\\
9: &\quad \text{if } x_6 = 0 \text{ goto } 14;\\
10: &\quad x_3 := x_3 * 10;\\
11: &\quad x_3 := x_3 + x_5;\\
12: &\quad x_6 := x_6 - 1;\\
13: &\quad \text{goto } 9;\\
14: &\quad x_4 := 0;\\
15: &\quad x_5 := 0;\\
16: &\quad x_6 := 0
\end{aligned}
$$

(1) Which value does P compute on input $x_1 = 2, x_2 = 3$?
Which value does P compute on input $x_1 = 3, x_2 = 0$?
Which value does P compute on input $x_1 = 15, x_2 = 2$?

(2) Which function $f : \mathbb{N}^2 \to \mathbb{N}$ is computed by P?

(3) Use the transformation presented in the lecture to construct a WHILE-IF program which has the same semantics as P.