Universität Koblenz-Landau

** **FB 4 Informatik**

Prof. Dr. Viorica Sofronie-Stokkermans
Dipl. Inform. Markus Bender

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Exercises for
"Advances in Theoretical Computer Science"
Exercise sheet **

1 Computability and (un)decidability

Exercise 1.1 (2+2+2 = 6p)

(1) State the theorem of Rice.

(2) Let $L_1 = \{n \mid L(M_n) \text{ is context-free}\}$ (where $L(M_n)$ is the language accepted by M_n). Prove that L_1 is undecidable using the theorem of Rice.

(3) Let B be a language in $\mathcal{L}_{0,\Sigma}$. Is $L_B = \{n \mid L(M_n) = B\}$ decidable? Justify your answer.

(4) Let $L_2 = \{n \mid L(M_n)$ contains all palindromes (possibly together with other words)$\}$ (where $L(M_n)$ is the language accepted by M_n).

Prove that L_2 is undecidable using the theorem of Rice.

Exercise 4.2 (2+1+3 = 6p)

(1) State the Post Correspondence Problem.

(2) Let $G = (\Sigma, R)$ be a semi-Thue system with $\Sigma = \{a, b\}$ and $R = \{ba \to a\}$.

(a) Show that $bbba \Rightarrow_G^* a$ by writing all the steps and underlining the occurrence of the left hand side of the rule in the current word at every step.

(b) Construct the correspondence system $P_{G,w',w''}$ as explained in the lecture, where $w' = bbba$ and $w'' = a$. Assume that rule 2 is $(X, Xw'X)$ and rule 3 is $(w''XX, X)$. Construct a solution for $P_{G,w',w''}$ with start 2 using the derivation $bbba \Rightarrow_G^* a$.

(3) Let $G = (\Sigma, R)$ be a semi-Thue system with $\Sigma = \{a, b\}$ and $R = \{bba \to ba\}$.

(a) Show that $bbba \Rightarrow_G^* ba$ by writing all the steps and underlining the occurrence of the left hand side of the rule in the current word at every step.

(b) Construct the correspondence system $P_{G,w',w''}$ as explained in the lecture, where $w' = bbba$ and $w'' = ba$. Assume that rule 2 is $(X, Xw'X)$ and rule 3 is $(w''XX, X)$. Construct a solution for $P_{G,w',w''}$ with start 2 using the derivation $bbba \Rightarrow_G^* ba$.