Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans
Dipl. Inf. Markus Bender

October 24, 2013

Exercises for
"Advances in Theoretical Computer Science"
Exercise sheet 1
Due on 29.10.13, 10:00

Exercise 1.1:
Get acquainted with the following definitions of Turing Machines and related concepts:

A *Turing Machine* (TM) $\mathcal{M}$ is a tuple $\mathcal{M} = (K, \Sigma, \delta, s)$ with

- K a finite set of states, $h \notin K$,
- Σ an alphabet, $L, R \notin \Sigma$ and $\# \in \Sigma$,
- $\delta : K \times \Sigma \to (K \cup \{h\}) \times (\Sigma \cup \{L, R\})$ a transition function, and
- $s \in K$ an initial state.

The transition $\delta(q, a) = (q', x)$ describes that if a TM is in state $q \in K$ and the symbol $a \in \Sigma$ is read, the TM changes its state to $q' \in K \cup \{h\}$ and

- moves the head one step to the left, iff $x = L$
- moves the head one step to the right, iff $x = R$
- does not move the head but prints the symbol $b \in \Sigma$ on the tape, iff $x = b \in \Sigma$

A *configuration* C of a TM $\mathcal{M} = (K, \Sigma, \delta, s)$ is a pair $C = q, w\underline{a}u$, with

- $q \in K \cup \{h\}$, the current state,
- $w \in \Sigma^*$, the tape contents left of the head,
- $a \in \Sigma$, the tape content under the head (the current symbol),
- $u \in \Sigma^*(\Sigma - \{\#\}) \cup \{\varepsilon\}$, the tape contents right of the head,

The *initial configuration* C_0 of $\mathcal{M}$ is defined as $C_0 = s, \#w\underline{\#}$ with *input* $w \in \Sigma^*$.

$C_2 = q_2, w_2\underline{a_2}u_2$ is a *successor configuration* of $C_1 = q_1, w_1\underline{a_1}u_1$, written as $C_1 \vdash_{\mathcal{M}} C_2$, iff there is a transition $\delta(q_1, a_1) = (q_2, b)$ and:

Case 1: $b \in \Sigma$**.** Then $w_1 = w_2, u_1 = u_2, a_2 = b$.

Case 2: $b = L$**.** Then for w_2 and $a_2 : w_1 = w_2a_2$. For u_2 : If $a_1 = \#$ and $u_1 = \varepsilon$, then $u_2 = \varepsilon$, otherwise $u_2 = a_1u_1$.

Case 3: $b = R$**.** Then for $w_2 = w_1a_1$. For a_2 and u_2 : If $u_1 = \varepsilon$, then $u_2 = \varepsilon$ and $a_2 = \#$, otherwise $u_1 = a_2u_2$.

$C_0 \vdash^*_{\mathcal{M}} C_n$ is called *computation*, iff for all C_i with $0 \le i < n$, C_{i+1} is a successor configuration of C_i.

Exercise 1.2:

a) Define a Turing Machine $\mathcal{M}_a$ that accepts all words $w \in \{|\}^*$ with an even length, i.e. $\mathcal{M}_a$ holds, iff w has even length, otherwise $\mathcal{M}_a$ does not terminate.

b) Define a Turing Machine $\mathcal{M}_d$ that decides if a word $w \in \{|\}^*$ has an even length.

   $s, \#w\underline{\#} \vdash^*_{\mathcal{M}_d} h, \#Y\underline{\#}$, iff w has even length,
   $s, \#w\underline{\#} \vdash^*_{\mathcal{M}_d} h, \#N\underline{\#}$, iff w has odd length.

c) Define a Turing Machine $\mathcal{M}_i$ that adds one $|$ to an input word $w \in \{|\}^*$.

   $s, \#w\underline{\#} \vdash^*_{\mathcal{M}_i} h, \#w|\underline{\#}$.

You can decide to give the formal definition of the Turing Machines or to draw it in the flow chart notation.

The submission of the solutions is not compulsory. If you want to submit your solutions, please do so until 29.10.13, 10:00 s.t.. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.
Submission possibilities:

- By e-mail to mbender@uni-koblenz.de with the keyword "Homework ACTCS" in the subject.
- Put it in the box in front of Room B 222.