Universität Koblenz-Landau

FB 4 Informatik

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Exercises for
"Advances in Theoretical Computer Science"
Exercise sheet 12

Exercise 12.1:
Prove that for every alphabet Σ with $|\Sigma| \geq 2$ it is undecidable whether for DCFL languages L_1, L_2 we have $L_1 \subseteq L_2$.

Hint: Reduction to the problem of testing emptiness for intersection of DFCL languages. Use the fact that the complement of a DCFL language is a DCFL language.

Exercise 12.2:
A map $h : \Sigma_1^* \to \Sigma_2^*$ is a monoid homomorphism if it has the property that for all words $w, w' \in \Sigma_1^*$, $h(w_1 w_2) = h(w_1)h(w_2)$.

Prove that the following problem is undecidable:

Let $f, g : \Sigma_1^* \to \Sigma_2^*$ be monoid homomorphisms. Assume that $\Sigma_1 = \{a_1, \dots, a_n\}$.

Is there a word $w \in \Sigma_1^*$ such that $f(w) = g(w)$?

Hint: Use the fact that the Post correspondence problem is undecidable. (Note that f and g are completely described by their values on $a_1, \dots, a_n$.)

Exercise 12.3:

Definitions. Assume we are in propositional logic with propositional variables Π.

- A *literal* L is a propositional variable P or the negation of a propositional variable $\neg P$.
- A propositional formula is in *disjunctive normal form (DNF)* if it has the form $(L_1^1 \wedge \dots \wedge L_{n_1}^1) \vee \dots \vee (L_1^m \wedge \dots \wedge L_{n_m}^m)$.
- A propositional formula is a *clause* if it is of the form $L_1 \vee \dots \vee L_n$ (i.e. is a disjunction of literals). A *Horn clause* is a clause which contains at most one positive literal. (For instance $P \vee \neg Q \vee \neg R$ and $\neg Q \vee \neg R$ are Horn clauses but $P \vee Q \vee \neg R$ is not a Horn clause.)

(1) Is it true that for every formula F in disjunctive normal form we can check whether F is satisfiable in polynomial time? Briefly justify your answer.

(2) Is it true that for every formula F which is a conjunction of Horn clauses we can check whether F is satisfiable in polynomial time? Briefly justify your answer.

Remark: For answering these questions you do not need to construct Turing machines. You can use results on propositional logic presented e.g. in the lecture "Logik für Informatiker" (e.g. the existence of algorithms for checking the satisfiability of sets (= conjunctions) of Horn clauses).

The submission of the solutions is not compulsory. If you want to submit your solutions, please do so until Tuesday, 28.1.2014, 10:00 s.t.. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.
Submission possibilities:

- By e-mail to mbender@uni-koblenz.de with the keyword "Homework ACTCS" in the subject.
- Put it in the box in front of Room B 222.