Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans
Dipl. Inform. Markus Bender

January 30, 2014

Exercises for
"Advances in Theoretical Computer Science"
Exercise sheet 13

Exercise 13.1:
Give a function $f : \Sigma^* \rightarrow \Sigma^*$ which polynomially reduces L_1 to L_2, or explain why this is not possible:

(1) $\Sigma = \{0, 1, 2\}$;
$L_1 = \{w \in \{0, 1\}^* \mid w$ is the binary representation of a prime number$\}$;
$L_2 = \{w \in \{0, 1, 2\}^* \mid w$ is the representation of a prime number in base 3$\}$.

(2) $\Sigma = \{0, 1\}, L_1$ as in (1);
$L_2 = \{w \in \{1\}^* \mid w$ is the representation of a prime number in base 1$\}$.

Exercise 13.2:
We know that SAT is NP-complete. In the previous exercise (28.01.14) we saw that satisfiability of formulae in DNF can be checked in polynomial time, so DNF-SAT $= \{F \mid F$ is a satisfiable formula of propositional logic in disjunctive normal form$\}$ is in P.

If we could construct a polynomial reduction of SAT to DNF-SAT (i.e. if we could prove that SAT $\prec_{\mathsf{pol}}$ DNF-SAT) then we could show that P = NP.

Formulae in propositional logic can be transformed to DNF using distributivity:

$$A \wedge (B_1 \vee \cdots \vee B_k) \equiv (A \wedge B_1) \vee \cdots \vee (A \wedge B_k).$$

Why does this not lead to a polynomial reduction?

Exercise 13.3:
Consider the following propositional logic formula:

$$F : \quad (P \vee \neg Q \vee \neg(R \vee \neg S)) \wedge (Q \vee \neg R \vee S)$$

Apply Steps 1-4 on page 47-49 of the slides from 30.01.2014 to this formula for computing the formula in 3-CNF associated to F (formula which is satisfiable iff F is satisfiable).

Exercise 13.4:

(1) Draw the complete graphs with $3, 4$ and 5 vertices.

(2) Consider the undirected graph $G = (V, E)$, where $V = \{a, b, c, d, e, f\}$ and

$$E = \{(a, b), (a, c), (a, e), (a, f), (b, c), (b, d), (b, e), (c, e), (c, f)\}.$$

(Note that in an undirected graph the edge (x, y) is identical to the edge (y, x), i.e. they are not ordered pairs but sets (or 2-multisets) of vertices.)

  (a) Draw the graph G.

  (b) Does G have a clique of size 3? Does G have a clique of size 4? Does G have a clique of size 5?

(3) Consider the following formula in 3-CNF:

$$F : \quad (\neg P_1 \vee P_2 \vee P_3) \wedge (P_1 \vee \neg P_2 \vee P_4) \wedge (P_2 \vee \neg P_3 \vee \neg P_4)$$

  (a) Is the formula satisfiable? If yes then give a satisfying assignment.

  (b) Starting from F construct the pair (G_F, k_F) as explained on the slides from 30.01.2014.

  (c) Has the graph G_F a clique of size k_F? If so indicate such a clique and reconstruct from it an assignment which makes F true.

Exercise 13.5:
Consider the following problem:

SET PACKING = $\{(C, l) \mid C = \{S_1, \dots, S_n\}$, every S_i is a finite set and there exists $D \subseteq C$ with l elements such that the elements of D are pairwise disjoint$\}$

(1) Prove that SET PACKING $\in$ NP.

For every pair (G, k), where $G = (V, E)$ is an undirected graph with vertices $\{v_1, \dots, v_m\}$ and edges in E we associate the pair (C, l), where $l = k$ and $C = \{S_1, \dots, S_m\}$, with $S_i = \{(v_i, v_j), (v_j, v_i) \mid (v_i, v_j) \notin E\}$.

(2) Estimate the time needed for constructing (C, l) from (G, k).

Prove:

(3) $S_i \cap S_j \neq \emptyset$ if and only if there is no edge between v_i and v_j in G.

(4) If G' is a clique of G with size k, with vertices $\{v_{i_1}, \dots, v_{i_k}\}$ then the sets in $D = \{S_{i_1}, \dots, S_{i_k}\}$ are pairwise disjoint.

(5) G hat a clique of size k iff there exists a subset D of C with l elements such that the elements of D are pairwise disjoint.

(6) Infer that Clique (the problem whether a graph has a clique of size k) can be polynomially reduced to SET PACKING.

(7) Is SET PACKING NP-complete? Justify your answer.

The submission of the solutions is not compulsory. If you want to submit your solutions, please do so until Tuesday, 4.2.2014, 10:00 s.t.. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.
Submission possibilities:

- By e-mail to mbender@uni-koblenz.de with the keyword "Homework ACTCS" in the subject.
- Put it in the box in front of Room B 222.