Universität Koblenz-Landau

FB 4 Informatik

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Exercises for
"Advances in Theoretical Computer Science"
Exercise sheet 2

Exercise 2.1:
Let $\Sigma = \{a, b\}$.

(a) Give descriptions as flow charts (diagrams) for deterministic Turing machines M_Y, M_N which for every $w \in \Sigma^*$ compute as follows:

$$s, \#w\underline{\#} \vdash_{M_Y} h, \#Y\underline{\#}$$

$$s, \#w\underline{\#} \vdash_{M_N} h, \#N\underline{\#}.$$

(b) Give a description as a flow chart (diagram) for a deterministic Turing Machine M_{pal} with two tapes which recognizes palindroms over Σ, i.e. it computes as follows:

$$\begin{array}{ll} s, \#w\underline{\#}, \underline{\#} \vdash^*_{M_{\mathsf{pal}}} h, \#Y\underline{\#}, \underline{\#} & \text{if } w \in \Sigma^* \text{ is a palindrom} \\ s, \#w\underline{\#}, \underline{\#} \vdash^*_{M_{\mathsf{pal}}} h, \#N\underline{\#}, \underline{\#} & \text{if } w \in \Sigma^* \text{ is not a palindrom.} \end{array}$$

You can use the DTM $L_\#$ and $R_\#$ presented in the lecture. You can introduce „auxiliary Turing machines", to ease the task — as long as you define what they are doing. In (b) you can for instance also use the DTM in (a).

Exercise 2.2:
Define a 3-tape Turing machine $\mathcal{M}_{mul}$ that calculates the product of the numbers n_1, n_2 in unary notation.

$$s, \#|^{n_1}\#|^{n_2}\underline{\#}, \underline{\#}, \underline{\#} \vdash^*_{\mathcal{M}_{mul}} h, \#|^{n_1 * n_2}\underline{\#}, \underline{\#}, \underline{\#}.$$

You can decide to give the formal definition of the Turing machines or to draw it in the flow chart (diagram) notation. You can introduce „auxiliary Turing machines", to ease the task — as long as you define what they are doing.

Exercise 2.3:
Define a Turing machine (you are free to choose any variant you like) that accepts the language $L = \{|^n \mid n \text{ is prime}\}$.

You can decide to give the formal definition of the Turing machines or to draw it in the flow chart (diagram) notation. You can introduce „auxiliary Turing Machines“, to ease the task — as long as you define what they are doing.

Exercise 2.4:
Prove or refute the following statements:

(a) The union of two decidable languages is decidable.

(b) The intersection of two decidable languages is decidable.

(c) The concatenation of two decidable languages is decidable.

(d) The complement of a decidable language is decidable.

(e) The union of two recursively enumerable languages is recursively enumerable.

(f) The intersection of two recursively enumerable languages is recursively enumerable.

(g) The concatenation of two recursively enumerable languages is recursively enumerable.

(h) The complement of a recursively enumerable language is recursively enumerable.

The submission of the solutions is not compulsory. If you want to submit your solutions, please do so until Tuesday, 5.11.2013, 10:00 s.t.. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.
Submission possibilities:

- By e-mail to mbender@uni-koblenz.de with the keyword “Homework ACTCS” in the subject.
- Put it in the box in front of Room B 222.