Universität Koblenz-Landau

FB 4 Informatik

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Exercises for
"Advances in Theoretical Computer Science"
Exercise sheet 3

Exercise 3.1:
Let L_1, L_2, L_3 be languages, where L_2 is recursively enumerable and L_3 is decidable.
Prove or refute the following statements:

1. If $L_1 \subseteq L_3$, then L_1 is decidable.
2. If $L_3 \subseteq L_1$, then L_1 is decidable.
3. If $L_1 \subseteq L_2$, then L_1 is recursively enumerable.
4. If $L_2 \subseteq L_1$, then L_1 is recursively enumerable.

Exercise 3.2:
Decide for each of the following problems if it is decidable or not. Justify your answer.

a) $P_1 := \{n \in \mathbb{N} \mid \mathcal{M}_n \text{ does not hold on empty input}\}$

b) $P_2 := \{n \in \mathbb{N} \mid L(\mathcal{M}_n) = \emptyset\}$

c) $P_3 := \{(m,n) \in \mathbb{N} \times \mathbb{N} \mid L(\mathcal{M}_m) \cap L(\mathcal{M}_n) = \emptyset\}$

d) $P_4 := \{(m,n) \in \mathbb{N} \times \mathbb{N} \mid L(\mathcal{M}_m) \subseteq L(\mathcal{M}_n)\}$

e) $P_5 := \{(n,w) \in \mathbb{N} \times \Sigma^* \mid$ For input w, $\mathcal{M}_n$ does not reach another configuration after $s, \#w\underline{\#}$ where the head is on a blank $(\#)\}$

f) $P_6 := \{(n,w,s) \in \mathbb{N} \times \Sigma^* \times \mathbb{N} \mid \mathcal{M}_n$ halts on input w after at most s steps $\}$

Remarks:

- $\mathcal{M}_n$ denotes the Turing machine with Gödel number n.
- $L(\mathcal{M})$ is the language accepted by the Turing machine $\mathcal{M}$ (i.e. the set of all words accepted by $\mathcal{M}$).

Hint: To prove undecidability you can for instance use properties of decidable languages (e.g. the fact that the complement of a decidable language is decidable) or a reduction to a problem which was already proved to be undecidable: (1) You are allowed to use the undecidability of the halting problem $HALT$, of H_0 or of K (notation as in the lecture); (2) if you have proven the (un-)decidability of P_i, you may use this result for any of the next tasks.

The submission of the solutions is not compulsory. If you want to submit your solutions, please do so until Tuesday, 12.11.2013, 10:00 s.t.. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.
Submission possibilities:

- By e-mail to mbender@uni-koblenz.de with the keyword “Homework ACTCS” in the subject.
- Put it in the box in front of Room B 222.