Universität Koblenz-Landau

FB 4 Informatik

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Exercises for
"Advances in Theoretical Computer Science"
Exercise sheet 5

Exercise 5.1:
Write a GOTO program which computes the function $q : \mathbb{N} \to \mathbb{N}$ defined for every $n \in \mathbb{N}$ by:

$$q(n) \text{ is the sum of the digits in } n.$$

Remark: You are allowed to use the following instructions:

$x_i := c$	$x_i := c$ op x_j	goto l	Here: x_i, x_j, x_k are registers
$x_i := x_j$	$x_i := x_j$ op c	if $x_i = 0$ goto l	c is a constant
	$x_i := x_j$ op x_k		op $\in \{+, -, *\}$
			and l is a label.

Exercise 5.2:
Let P be the following GOTO program:

```
1: x4 := x1;

2: if x4 = 0 goto 10;
3: x5 := x2;

4: if x5 = 0 goto 8;
5: x3 := x3 + 1;
6: x5 := x5 - 1;
7: if x6 = 0 goto 4;

8: x4 := x4 - 1;
9: if x6 = 0 goto 2;

10: x5 := x5 - 1
```

(1) Which value does P compute on the following inputs:
   (a) $x_1 = 2, x_2 = 0$;
   (b) $x_1 = 0, x_2 = 3$;
   (c) $x_1 = 2, x_2 = 3$

(2) Which function $f : \mathbb{N}^2 \to \mathbb{N}$ is computed by P?

(3) Use the transformation presented in the lecture to construct a WHILE-IF program with the same semantics as P.

Exercise 5.3:
Let P be the following WHILE program:

$x_4 := 10 - x_1;$
$x_5 := 1 - x_4;$
while $x_5 \neq 0$ do
    $x_5 := x_5 + 1$
end;
$x_4 := x_2 - 1;$
$x_5 := x_1;$
$x_3 := x_1;$
while $x_4 \neq 0$ do
    $x_5 := x_5 * 10$
    $x_3 := x_3 + x_5$
    $x_4 := x_4 - 1$
end;
$x_5 := 0$

(1) Which value does P compute on input $x_1 = 2, x_2 = 3$? Which value does P compute on input $x_1 = 3, x_2 = 0$?

(2) Which function $f : \mathbb{N}^2 \to \mathbb{N}$ is computed by P?

(3) Use the transformation presented in the lecture to construct a GOTO program which has the same semantics as P.

Exercise 5.4:

(1) Let $f : \mathbb{N} \to \mathbb{N}$ be a bijective function which is WHILE computable. Show that its inverse, $f^{-1} : \mathbb{N} \to \mathbb{N}$, is WHILE computable as well.
Can we find any GOTO computable bijection $g : \mathbb{N} \to \mathbb{N}$ for which $g^{-1} : \mathbb{N} \to \mathbb{N}$ is not GOTO computable?

(2) Let $f : \mathbb{N} \times \mathbb{N}$ be a bijective function which is WHILE computable. Let P be the WHILE program which computes f. Write a WHILE program, which uses P, with the property that started with input n_1 in register x_1 it ends with value n_2 in register x_2 and value n_3 in register x_3, where n_2 and n_3 are such that $f(n_2, n_3) = n_1$.

Remark: You are allowed to use the following WHILE programs and constructions:

$x_i := c$	$x_i := c$ op x_j	$P_1; P_2$	Here: x_i, x_j, x_k are registers
$x_i := x_j$	$x_i := x_j$ op c	while $x_i \neq 0$ do P_1 end	c is a constant
	$x_i := x_j$ op x_k	if $x_i = 0$ then P_1 end	op $\in \{+, -, *\}$
			and P_1, P_2 are WHILE programs.

The submission of the solutions is not compulsory. If you want to submit your solutions, please do so until Tuesday, 26.11.2013, 10:00 s.t.. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.
Submission possibilities:

- By e-mail to mbender@uni-koblenz.de with the keyword "Homework ACTCS" in the subject.
- Put it in the box in front of Room B 222.