Universität Koblenz-Landau

FB 4 Informatik

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Exercises for
“Advances in Theoretical Computer Science”
Exercise sheet 6

Exercise 6.1:
Let P be the following WHILE program:

$x_3 := 0$; $x_4 := x_2$;
while $x_1 \neq 0$ do
  $x_2 := x_4$;
  while $x_2 \neq 0$ do
    $x_3 := x_3 + 1$; $x_2 := x_2 - 1$
  end; $x_1 := x_1 - 1$
end

Find a WHILE-IF program P' with one WHILE loop only which has the same semantics as P (i.e. $\Delta(P') = \Delta(P)$). Use for this the results on the slides from 21.11.2013.

Exercise 6.2:
Prove that the following functions are primitive recursive:

(1) $c_s^k : \mathbb{N}^k \to \mathbb{N}$, where $s \in \mathbb{N}$, defined for every $\mathbf{n} \in \mathbb{N}^k$ by: $c_s^k(\mathbf{n}) = s$.

(2) $\mathsf{fac} : \mathbb{N} \to \mathbb{N}$, defined for every $n \in \mathbb{N}$ by: $\mathsf{fac}(n) = n!$.

(3) $\mathsf{exp} : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$, defined for every $(n, m) \in \mathbb{N} \times \mathbb{N}$ by: $\mathsf{exp}(n, m) = n^m$.

(4) $\mathsf{eq} : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$, defined for every $(n, m) \in \mathbb{N} \times \mathbb{N}$ by: $\mathsf{eq}(n, m) = \begin{cases} 1 & \text{if } x = y \\ 0 & \text{otherwise} \end{cases}$.

Remark: You are allowed to use all primitive recursive functions introduced in the lecture.

Exercise 6.3:
Consider the following primitive recursive functions:

- $f_1 = + \circ (- \circ (\pi_1^2, c_5^2), * \circ (\pi_2^2, \pi_2^2))$
- $f_2 = \mathcal{PR}[(+1) \circ 0, * \circ ((+1) \circ \pi_1^2, \pi_2^2)]$
- $f_3 = \mathcal{PR}[c_1^1, * \circ (\pi_1^3, \pi_3^3)] \circ ((-1) \circ \pi_1^2, \pi_2^2)$

(a) Which is the arity of f_1, of f_2 and of f_3? (i.e. how many arguments does each of these function have?)

(b) What do these functions compute if all arguments are equal to 2?

(c) What do these functions compute in general?

Note: We used the following notation (cf. also slides from 28.11.2013):

- $\circ$ (function composition) is defined as in the lecture: $(g \circ (h_1, \dots, h_r))(\mathbf{n}) = g(h_1(\mathbf{n}), \dots, h_r(\mathbf{n}))$.
- if $j \leq k$, π_j^k is the projection function defined as in the lecture: $\pi_j^k(n_1, \dots, n_k) = n_j$.
- $(+1) : \mathbb{N} \to \mathbb{N}$ is defined as in the lecture, by: $(+1)(n) = n + 1$.
- $(-1) : \mathbb{N} \to \mathbb{N}$ is defined by: $(-1)(n) = \begin{cases} 0 & \text{if } n = 0 \\ n - 1 & \text{otherwise} \end{cases}$.
- $* : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ is defined for all $n_1, n_2 \in \mathbb{N}$ by: $*(n_1, n_2) = n_1 * n_2$.
- $+ : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ is defined for all $n_1, n_2 \in \mathbb{N}$ by: $+(n_1, n_2) = n_1 + n_2$.
- $- : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ is defined for all $n_1, n_2 \in \mathbb{N}$ by: $-(n_1, n_2) = n_1 - n_2$.
- for all $s, k \in \mathbb{N}$, $c_s^k : \mathbb{N}^k \to \mathbb{N}$ is defined for all $\mathbf{n} \in \mathbb{N}^k$ by: $c_s^k(\mathbf{n}) = s$.

The submission of the solutions is not compulsory. If you want to submit your solutions, please do so until Tuesday, 3.12.2013, 10:00 s.t.. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.
Submission possibilities:

- By e-mail to mbender@uni-koblenz.de with the keyword "Homework ACTCS" in the subject.
- Put it in the box in front of Room B 222.