Advanced Topics in Theoretical Computer Science

Part 2: Register machines

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- Recapitulation: Turing machines and Turing computability
- Register machines (LOOP, WHILE, GOTO)
- Recursive functions
- The Church-Turing Thesis
- Computability and (Un-)decidability
- Complexity

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2. Register Machines

- Register machines (Random access machines)
- LOOP Programs
- WHILE Programs
- GOTO Programs
- Relationships between LOOP, WHILE, GOTO
- Relationships between register machines and Turing machines

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Register Machines

The register machine gets its name from its one or more "registers":

In place of a Turing machine's tape and head (or tapes and heads) the model uses multiple, uniquely-addressed registers, each of which holds a single positive integer.

Register Machines

In comparison to Turing machines:

- equally powerful fundament for computability theory
- **Advantage:** Programs are easier to understand

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similar to ...

the imperative kernel of programming languages

pseudo-code

Register Machines

Computation of a mod b (pseudocode)

$r := a$;

while $r \geq b$ do

    $r := r - b$

end;

return r

Register Machines

Definition: Questions

Which instructions (if, while, goto?)

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- **Data types:**
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- **Data structures**
  - Unbounded but finite number of registers denoted $x_1, x_2, x_3 \ldots, x_n$;
    each register contains a natural number
    (no arrays, objects, ...)

Register Machines

Settings (Informally)

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  - Increment/Decrement a register

Register Machines

Settings (Informally)

- **Atomic instructions:**
  - Increment/Decrement a register
- **Input/Output**
  - **Input:** n input values in the first n registers
    All the other registers are 0 at the beginning.
  - **Output:** In register $n + 1$.

Example: LOOP Programs

Syntax

Definition

- **Atomic programs:** For each register x_i:
  - $x_i := x_i + 1$
  - $x_i := x_i - 1$

  are LOOP instructions and also LOOP programs.

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- If P is a LOOP program then
  - loop x_i do P end is a LOOP instruction and a LOOP program.

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  - loop x_i do P end is a LOOP program (and a LOOP instruction)

Example: WHILE Programs

Syntax

Definition

- **Atomic programs:** For each register x_i:
  - $x_i := x_i + 1$
  - $x_i := x_i - 1$

  are WHILE instructions and also WHILE programs.
- If P_1, P_2 are WHILE programs then
  - $P_1; P_2$ is a WHILE program
- If P is a WHILE program then
  - while $x_i \neq 0$ do P end is a WHILE program (and a WHILE instruction)

Example: GOTO Programs

Syntax Indexes (numbers for the lines in the program) $j \geq 0$

Definition

- **Atomic programs:**
  - $x_i := x_i + 1$
  - $x_i := x_i - 1$

  are GOTO instructions for each register x_i.
- If x_i is a register and j is an index then
  - if $x_i = 0$ goto j is a GOTO instruction.
- If $I_1, \ldots, I_k$ are GOTO instructions and $j_1, \ldots, j_k$ are indices then
  - $j_1 : I_1; \ldots; j_k : I_k$ is a GOTO program

Register Machines

Definition

A register machine is a machine consisting of the following elements:

- A finite (but unbounded) number of registers $x_1, x_2, x_3 \ldots, x_n$; each register contains a natural number.
- A LOOP-, WHILE- or GOTO-program.

Register Machines: State

Definition (State of a register machine)

The state s of a register machine is a map:

$$s : \{x_i \mid i \in \mathbb{N}\} \to \mathbb{N}$$

which associates with every register a natural number as value.

Register Machines: State

Definition (Initial state; Input)

Let $m_1, \ldots, m_k \in \mathbb{N}$ be given as input to a register machine.

In the input state s_0 we have

- $s_0(x_i) = m_i$ for all $1 \leq i \leq k$
- $s_0(x_i) = 0$ for all $i > k$

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Definition (Output)

If a register machine started with the input $m_1, \ldots, m_k \in \mathbb{N}$
halts in a state s_{term} then:

$$s_{\text{term}}(x_{k+1})$$

is the output of the machine.