



Algorithm 1: Algorithm of Klein to compute min cost flows

$\text{MINCOSTFLOW-KLEIN}(G, l, u, b, c)$

Input : A simple directed graph $G = (V, R, \alpha, \omega)$ with capacity functions $0 \leq l(r) \leq u(r)$ for all $r \in R$, desired excess values $b: V \rightarrow \mathbb{R}$ and flow costs $c: R \rightarrow \mathbb{R}_+$.

Output: A minimum cost b -flow f . If l, u and c are integral, so is f .

- 1 Compute a feasible b -flow f // Can be done by computing a maximum flow
 - 2 **while** the residual network G_f contains a negative cycle C **do**
 - 3 Let $\Delta := \min_{\sigma r \in C} c(\sigma r)$ be the minimal residual capacity along C
 - 4 Augment f along C by Δ // Eliminate the negative cycle C
 - 5 **return** f
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