Exercise Sheet 6

To be done in the tutorials on 28.05.2018 & 01.06.2018

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In-Class Exercises

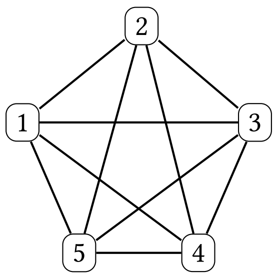
(To be done in the tutorials on 28.05.2018 & 01.06.2018)

Exercise 6.1

Let $G = (V, E)$ be a graph with no double edges and $B \in \mathbb{R}^{|V|\times|E|}$ its incidence matrix. Which meaning do the entries of the matrix product $BB^\top$ have, where $B^\top$ denotes the transposed matrix of B?

Exercise 6.2

Consider the following graph:



a) Determine the adjacency matrix $\mathbf{A}$ and the incidence matrix $\mathbf{B}$ to this graph.

b) Compute the product $\mathbf{A} \cdot \mathbf{A} =: \mathbf{A}^2$ of the adjacency matrix. Which meaning do the entries of this matrix have?

Exercise 6.3

Prove the following statement:
Let $G = (V, R, \alpha, \omega)$ be a finite directed graph. Then it holds that the number of vertices with odd degree is even.

Exercise 6.4

Let $G = (V, R, \alpha, \omega)$ be a directed graph. To every edge $r \in R$, we define the inverse edge r^{-1} via

$$\alpha(r^{-1}) := \omega(r) \text{ and } \omega(r^{-1}) := \alpha(r).$$

In this exercise, we use adjacency matrices for storing graphs. Find an algorithm that gets G as an input and computes G^{-1}. Determine the runtime of your algorithm.

Exercise 6.5

Let the adjacency matrix A to a directed graph G be given and let $v \in V$. Find an algorithm that computes the outer degree of v.