QUERYING FOR DIFFERENT SIMILAR IMAGES USING MAXIMUM INDEPENDENT SETS OF A GRAPH

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Abstract

In a tool dedicated to image retrieval, the system is often in charge to propose the user a selection of images close to his/her query. This paper is intended to present a new method based on graph theory techniques in order to select such a set of images among those that might be offered.
Keywords: *Image retrieval, graph theory, maximal independent sets, cliques*

1 Introduction

In image retrieval systems, images are generally internally represented by a vector of features. When an user makes a query with an image as example to the system, the vector of features of the querying image is compared to those existing in the database. This comparison is most of the time made by a weighted distance. The set of images that have the closest features according to the distance to the query are assumed to correspond to the user wish and are extracted by the system. The images in such a set are sorted and displayed with decreasing similarity degrees.

Only a small number of images is shown to the end-user. The fact is that, in many databases, some images may have features very close one to the other because of the closeness of the images themselves. Imagine for instance a series of photographic pictures of different objects constituted such that an object is taken every five degrees (for instance the well-known COIL database). The expectation of the user might be to find images that are close to his/her query without having firstly to look about thirty very close images.

From the point of view of the organization of images in the image space, images are grouped in high densities clusters. Therefore displaying the k-nearest neighbors of the query may not be relevant and will leave no choice to the user but modifying the original query (even if the query is well formulated), or ask the system to display the next k-nearest neighbors. In the latter, in worst cases, it may happen that the same problem may occur: the system may display again a series of k very close images (see fig. 1). Note that the same problem may arise for any image database whose structure is a priori not known.

The aim of that paper is to provide an alternative to the k-nearest neighbors method of images selection in using an aspect of graph theory: the maximal independent sets. Such approaches have already been given []. They mainly focus on giving an aspect of vector space to the space of features and use

2 Graph Theory Background

The following definitions come from [1]. Given a set V, we will denote by $V^{(2)}$ the set of unordered pairs of V. A *graph* G is an ordered pair (V, E) such that E is a subset of $V^{(2)}$. If $\{x, y\}$ is an element of E, x and y are *adjacent*. The elements of V are called *vertices*, those of E *edges*. A graph $G' = (V', E')$ is a *subgraph* of G if $V' \subset V$ and $E' \subset E$. A graph is *complete* if $E = V^{(2)}$. A complete maximal (in the sense of inclusion) subgraph of G is called a *clique* of G.

In general, the complement of G is the graph $\bar{G} = (V, V^{(2)} \setminus E)$. A set of vertices is *independent* if no two elements of it are adjacent. As for cliques, a maximal independent set of vertices (MIS) is a set of vertices in which adding a vertex to the set does not preserve the property of independency anymore. It is easy to show that a clique in a given graph corresponds to a MIS in its complement [2] (see fig. 2).

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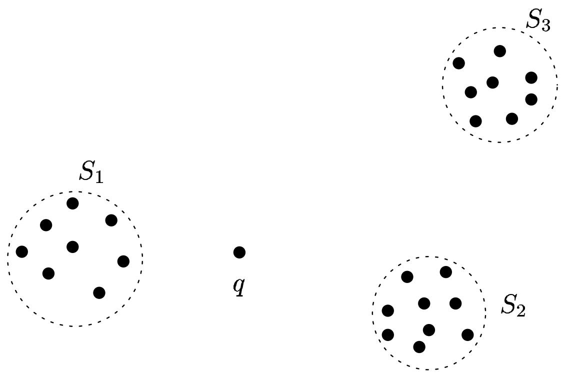

Figure 1: A subset of the first 25 images of the database closest to the query image q. We assume here that the feature space is bidimensional and that the similarity measure is the Euclidian distance. If the retrieval system is supposed to give the eight nearest images, the set S_1 is returned. Problems may appear if images of the database are organized in dense clusters because they are "too" close one to the other. Asking for the next eight nearest images does not solve that problem.

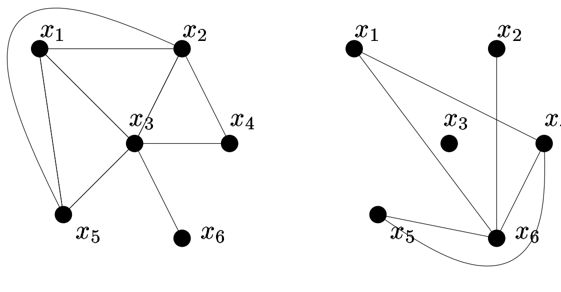

Figure 2: A graph and its complement. The set of its cliques is $\{\{x_1, x_2, x_3, x_5\}, \{x_2, x_3, x_4\}, \{x_3, x_6\}\}$, The set of its MIS $\{\{x_1, x_4, x_6\}, \{x_2, x_6\}, \{x_3\}, \{x_4, x_5, x_6\}\}$.

Enumerating the whole set of cliques (or and this is computationally equivalent the set of MIS) of a graph is a problem being studied since the late 50s [3]. The best exact algorithm known so far is due to Tomita and al. [4] and has a worst complexity of $O(\frac{n}{3})$ if n denotes the number of vertices in a graph. This is precisely the largest number of cliques that might appear in a graph [5].

To each vertex x of V may be associated its *neighborhood* $N(x)$ defined as the set of vertices adjacent to x:

$$(\forall x \in V) \quad N(x) = \Big\{ y \in V \text{ such as } \{x, y\} \in E \Big\}$$

Note that with our definition of a graph, a loop, that is, an edge whose extremities are the same vertex, cannot appear. Therefore, the neighborhood of a vertex does not contain that vertex.

At last, if an application w is defined from the set of edges of a graph (V, E) to a set Y, the graph is said to be *valuated.*

3 Features Space Structure

In the field of image retrieval, an image is represented by a set of *features*, classically consisting in a vector of real parameters. The features of two images are compared thanks to a similarity measure s which may be in some cases a distance. The *features space* $\mathcal{F}$ is built according to s. Two images are considered to be similar if their similarity measure is less than a given real threshold ε.

As the number of images in the image database is finite, the features space may be considered as a graph whose vertices are the images themselves. There is a priori an edge between any pair of different vertices: the graph is therefore complete. However, in valuating the graph by the similarity measure, it becomes possible to remove some of the edges if their corresponding weight is less than ε.

Considering the neighborhood of an vertex in that graph, it is easy to build the set of images that are close to that image-vertex. However, if we consider the example of 3, it is in general not possible to return to the user the whole set of these neighbors as such a set may be too large.

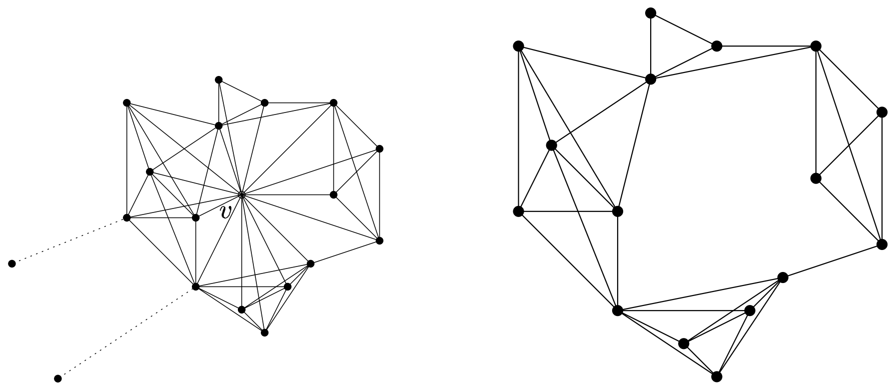

Figure 3: An example of the local structure of the features space. In the left part of the figure, the vertex v is linked by edges to its neighbors by solid lines. Vertices that are exterior to its neighborhood may however been linked to vertices of the neighborhood (shown in dotted lines). In the right part, only the neighborhood of v remains and will be used to select the set of most different similar images.

The classical approach consists in retrieving the k-nearest neighbors. Our approach is slightly different in the sense that we chose to extract the set of images similar to v in that neighborhood that will be as different as possible one from the other. In terms of graph theory, such a set can be considered as a maximal independent set in the neighborhood of v.

4 Maximum MIS vs Maximum Weighted MIS

A first approach may consist in extracting the set of maximum MIS, that is, the set $\mathcal{M}$ of maximal independent sets that have maximum cardinality. Several problems may arise from that approach. Generally speaking, there is no uniqueness of one maximum MIS and therefore the system will have to select the MIS that will be chosen as the correct one. The simplest and the most natural solution consists in selecting the maximum MIS that has the maximum weight.

Even if it has not been used in our vertices selection process, there is still a measure of similarity between any two pair of vertices in the graph. For any subset S of vertices in the graph, we can compute its associated weight according to the similarity measure s:

$$w(S) = \sum_{\{x,y\} \in S} s\Big(\{x, y\}\Big)$$

There is however no guarantee of the existence of one unique maximum in the set of maximum weighted MIS.

Moreover, a more constraining problem may arise if the graph in which the MIS are sought is the complement of multipartite complete graph (see fig. 4). In such a case, the number of MIS is exponential with respect to the number of vertices in the graph. Evaluating the set of all MIS and their weights might be, generally speaking, not computationally efficient (even if the computation is made off-line).

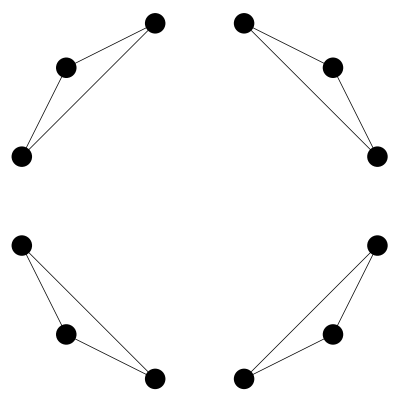

Figure 4: Illustration of a worst-case to find maximum MIS in a graph. The graph of 12 vertices consists in 4 disconnected triangles. The set of MIS consists in $81 = 3^4$ MIS. These MIS are also maximum.

An other problem may consist in the fact that a maximum MIS that has a maximum weight (in the set of maximum MIS) may not have the maximum weight of MIS that can be extracted from all the graph as the figure 5 shows it.

5 Practical considerations. Conclusion

In an image retrieval system, it may be interesting to look for a set of images that are close to a query but are as far one from the other. Such an approach may be considered under the viewpoint of graph theory and is equivalent to the search of MIS in a graph. It therefore gives an alternative means of selecting images from a set of similar ones. We presented here a concise description of some of the problems that may arise when looking for maximal independent sets in an image database if it is considered as a weighted graph. We are currently running experiments concerning the extraction of such sets in different images databases for different similarity measures. Two main further interesting facts may be seen in such an approach.

Image retrieval systems try now to integrate feedbacks from the user. It may occur that this one wishes not to see an image more. In such a case modifying existing classical method relying on analysis or algebra based methods is very constraining [6]. In our approach, it will

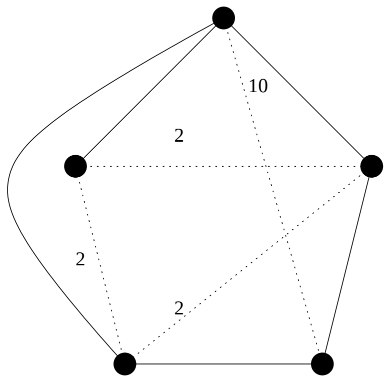

Figure 5: In such a graph, there are three MIS (shown in dotted lines). The weight of each edge is shown near the corresponding one. The maximum MIS has a weight of 6. The MIS that has the maximum weight is made of two vertices and has a weight of 10.

simply consists in removing a vertex and its neighbors from the graph to take into account the user's choice.

The second aspect concerns the main drawback of the method: its computational complexity even if the structure of the graph and the set of MIS in the neighborhood of each vertex can be computed off-line. The complexity of the search relies both on the size and the density (the ratio of the number of edges in the graph compared to the number of edges in the complete graph with the same number of vertices) of the graph. Evaluating that complexity is not a priori feasible except for very special classes of graphs. It might therefore be interesting not to define an absolute threshold for the whole features space but compute locally a threshold, for each vertex such that looking for a MIS in the corresponding graph might be easy from the computational point of view, that is, that does not exceed some given time constraint.

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