Probability Propagation Nets

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Abstract—A class of high level Petri nets, called "probability propagation nets", is introduced which is particularly useful for modeling probability and evidence propagation. These nets themselves are well suited to represent the probabilistic Horn abduction, whereas specific foldings of them will be used for representing the flows of probabilities and likelihoods in Bayesian networks.

Index Terms—Bayes procedures, Horn clauses, Petri nets, Probability, Propagation, Stochastic logic.

I. INTRODUCTION

This paper deals with the propagation of probabilities in Petri nets (PNs). That means, first of all, it is a paper about PNs and their ability to represent the dynamic in logical-probabilistic structures.

By far most of the papers about PNs and probabilities are about transitions whose duration is governed by a probability distribution. In contrast to that, we will introduce a class of PNs, called "Probability Propagation Nets" (PPNs), for developing transparent and well structured models in which probabilities are propagated, for example as decision aids or degrees of risk.

We will try out the modeling power of our approach by means of the probabilistic Horn abduction [1]–[3] and Bayesian networks (BNs) [4]. In doing so, we will avoid to give the impression that we are going to improve these approaches. However, using PNs means to work with one of the most famous modeling tools. So the outcome might have degrees of risk.

Different from some existing approaches on combining BNs and PNs (e.g. [5], [6]), we introduce step by step new PNs that are particularly well suited for our intentions; they are transparent and structured.

First, we modify the p/t-nets for representing logical inference [7] by inscribing tokens and arcs with probabilities. These nets, the PPNs, allow to represent stochastic inference and probabilistic Horn abduction.

Second, foldings of these nets reduce the net structure and allow the representation of BNs. Fortunately, the inscriptions and the firing rule remain clear.

In spite of the considerable complexity, the PNs are of manageable size. A particular advantage is that all the propagation processes are represented by t-invariants (because of the reproducibility of the propagations) and that the t-invariants can be calculated in the underlying p/t-nets.

Our paper is organized as follows. Section II comprises that part of PN theory which is needed for the subsequent sections. In section III PPNs are introduced on the basis of a PN representation of Horn formulas. The probabilistic Horn abduction is used to exemplify the modeling power of PPNs. Section IV addresses higher PPNs as foldings of PPNs. Here, BNs are used to demonstrate the modeling ability. This is continued in section V by means of popular examples. Section VI and an appendix conclude the paper.

II. PRELIMINARIES

Definition 1 1) A place/transition net (p/t-net) is a quadruple \( N = (S, T, F, W) \) where

\[ S \] and \( T \) are finite, non empty, and disjoint sets. \( S \) is the set of places (in the figures represented by circles), \( T \) is the set of transitions (in the figures represented by boxes).

(b) \( F \subseteq (S \times T) \cup (T \times S) \) is the set of directed arcs.

(c) \( W : F \to \mathbb{N} \setminus \{0\} \) assigns a weight to every arc. In case of \( W : F \to \{1\} \), we will write \( N = (S, T, F) \) as an abridgment.

2) The preset (postset) of a node \( x \in S \cup T \) is defined as 

\( \{y \in S \cup T | (y, x) \in F\} \) or \( \{y \in S \cup T | (x, y) \in F\} \).

The preset (postset) of a set \( H \subseteq S \cup T \) is \( \{x \in H | x \in S \cup T \} \).

For all \( x \in S \cup T \) it is assumed that \(|x| + |x| \geq 1\) holds; i.e. there are no isolated nodes.

3) A place \( p \) (transition \( t \)) is shared iff \( |p| \geq 2 \) or \( |t| \geq 2 \)

\((|p| \geq 2 \vee |t| \geq 2)\).

4) A place \( p \) is an input (output) boundary place iff \( p = \emptyset \) \((p = \emptyset)\).

5) A transition \( t \) is an input (output) boundary transition iff 

\( t = \emptyset \) \((t = \emptyset)\).

Definition 2 Let \( N = (S, T, F, W) \) be a p/t-net.

1) A marking of \( N \) is a mapping \( M : S \to \mathbb{N} \). \( M(p) \) indicates the number of tokens on \( p \) under \( M \). \( p \in S \) is marked by \( M \) iff \( M(p) \geq 1 \). \( H \subseteq S \) is marked by \( M \) iff at least one place \( p \in H \) is marked by \( M \). Otherwise \( p \) and \( H \) are unmarked, respectively.

2) A transition \( t \in T \) is enabled by \( M \), in symbols \( M(t) \), iff 

\[ \forall p \in t : M(p) \geq W((p, t)). \]

3) If \( M(t) \), the transition \( t \) may fire or occur, thus leading...
to a new marking $M'$, in symbols $M[t]M'$, with

$$M'(p) := \begin{cases} 
M(p) - W((p, t)) & \text{if } p \in \mathcal{U} \setminus \mathcal{T} \\
M(p) + W((t, p)) & \text{if } p \in \mathcal{P} \setminus \mathcal{T} \\
M(p) - W((p, t)) + W((t, p)) & \text{if } p \in \mathcal{T} \setminus \mathcal{U} \\
0 & \text{otherwise}
\end{cases}$$

for all $p \in \mathcal{S}$.

4) The set of all markings reachable from a marking $M_0$, in symbols $[M_0]$, is the smallest set such that

- $M_0 \in [M_0]$,
- $M \in [M_0] \land M[t]M' \Rightarrow M' \in [M_0]$.

$[M_0]$ is also called the set of follower markings of $M_0$.

5) $\sigma = t_1 \ldots t_n$ is a firing sequence or occurrence sequence for transitions $t_1, \ldots, t_n \in T$ iff there exist markings $M_0, M_1, \ldots, M_n$ such that

$$M_0[t_1]M_1[t_2] \ldots [t_n]M_n \text{ holds;}$$

in short $M_0[\sigma]M_n$. $M_0[\sigma]$ denotes that $\sigma$ starts from $M_0$. The firing count $\sigma(t)$ of $t$ in $\sigma$ indicates how often $t$ occurs in $\sigma$. The (column) vector of firing counts is denoted by $\sigma$.

6) The pair $(N, M_0)$ for some marking $M_0$ of $N$ is a p/t-net-system or a marked p/t-net. $M_0$ is the initial marking.

7) A marking $M \in [M_0]$ is reproducible iff there exists a marking $M' \in [M]', M' \neq M$ s.t. $M \in [M']$.

8) Moreover, the p-column-vector $0$ stands for the empty marking. A p/t-net is $0$-reproducing iff there exists a firing sequence $\varphi$ such that $0[\varphi]0$. A transition $t$ is $0$-fireable iff $t$ can be enabled by some follower marking of $0$.

**Definition 3** Let $N = (S, T, F, W)$ be a p/t-net;

1) $N$ is pure iff $\exists (x, y) \in (S \times T) \cup (T \times S) : (x, y) \in F \land (y, x) \in F$.

2) A place vector $(S)$-vector is a column vector $v : S \rightarrow \mathbb{Z}$ indexed by $S$.

3) A transition vector $(T)$-vector is a column vector $\omega : T \rightarrow \mathbb{Z}$ indexed by $T$.

4) The incidence matrix of $N$ is a matrix $[N] : S \times T \rightarrow \mathbb{Z}$ indexed by $S$ and $T$ such that

$$[N](p, t) = \begin{cases} 
-W((p, t)) & \text{if } p \in \mathcal{U} \setminus \mathcal{T} \\
W((t, p)) & \text{if } p \in \mathcal{P} \setminus \mathcal{T} \\
-W((p, t)) + W((t, p)) & \text{if } p \in \mathcal{T} \setminus \mathcal{U} \\
0 & \text{otherwise}
\end{cases}$$

$v^t$ and $A^t$ are the transposes of a vector $v$ and a matrix $A$, respectively. The columns of $[N]$ are $S$-vectors, the rows of $[N]$ are transposes of $T$-vectors. Markings are representable as $S$-vectors, firing count vectors as $T$-vectors.

**Definition 4** Let $I$ be a place invariant and $J$ a transition invariant of $N = (S, T, F, W)$.

1) $I$ is a place invariant (p-invariant) iff $I \neq 0$ and $I^t \cdot [N] = 0$.

2) $J$ is a transition invariant (t-invariant) iff $J \neq 0$ and $[N] \cdot J = 0$.

3) $\{p \in S \mid I(p) \neq 0\}$ and $\{t \in T \mid J(t) \neq 0\}$ are the supports of $I$ and $J$, respectively.

4) A p-invariant $I$ (t-invariant $J$) is

- non-negative iff $\forall p \in S : I(p) \geq 0 \forall t \in T : J(t) \geq 0$.
- positive iff $\forall p \in S : I(p) > 0 \forall t \in T : J(t) > 0$.
- minimal iff $I(J)$ is non-negative and $\exists$ p-invariant $I' : \|I'\| \leq \|I\|$ (and the greatest common divisor of all entries of $I(J)$ is 1).

5) The net representation $N_1 = (S_1, T_1, F_1, W_1)$ of a p-invariant $I$ is defined by

$$S_1 := \|I\|$$

$$T_1 := S_1 \cup S_1$$

$$F_1 := F \cap ((S_1 \times T_1) \cup (T_1 \times S_1))$$

$$W_1$$

is the restriction of $W$ to $F_1$.

6) The net representation $N_J = (S_J, T_J, F_J, W_J)$ of a t-invariant $J$ is defined by

$$T_J := \|J\|$$

$$S_J := T_J \cup T_J$$

$$F_J := F \cap ((S_J \times T_J) \cup (T_J \times S_J))$$

$$W_J$$

is the restriction of $W$ to $F_J$.

7) $N$ is covered by a p-invariant $I$ (t-invariant $J$) iff $\forall p \in S : I(p) \neq 0 \forall t \in T : J(t) \neq 0$.

**Proposition 1** Let $(N, M_0)$ be a p/t-system, $I$ a p-invariant; then

$$\forall M \in [M_0] : I^t \cdot M = I^t \cdot M_0.$$
Definition 6 (Natural Multiset) Let $A$ be a non-empty set:

- $m : A \rightarrow \mathbb{N}$ is a natural multiset over $A$.
- $M(A)$ is the set of all natural multisets over $A$.

III. PROBABILITY PROPAGATION NETS

In this section, we introduce probability propagation nets (PPNs) which are a "probabilistic extension" of place/transition nets representing logical formulas (see [7], [8]). Starting with the canonical net representation of Horn formulas in conjunctive normal form, we enrich these formulas by probabilities as in probabilistic Horn abduction [1]–[3]. After that, we introduce an appropriate extension of the canonical (Petri) net representation modeling Horn formulas. The resulting PNs are called "probability propagation nets". The transformation of a logical Horn formula into the canonical net representation is detailedly described in [7]. In order to give a short summary and to introduce the relevant terms, we stick to an example.

Definition 7 Let $\tau = \neg a_1 \lor \cdots \lor \neg a_m \lor b_1 \lor \cdots \lor b_n$ be a clause;

in set notation: $\tau = \neg A \cup B$ for $\neg A = \{\neg a_1, \ldots, \neg a_m\}$ and $B = \{b_1, \ldots, b_n\}$;

- $\tau$ is a fact clause iff $\neg A = \emptyset$;
- $\tau$ is a goal clause iff $B = \emptyset$;
- $\tau$ is a rule clause iff $\neg A \neq \emptyset \land B \neq \emptyset$;
- $\tau$ is a Horn clause iff $|B| \leq 1$.

Let $\alpha$ be a conjunction of clauses, i.e. $\alpha$ is a conjunctive normal form (CNF) formula;

- $\mathcal{A}(\alpha)$ denotes the set of atoms of $\alpha$;
- $\mathcal{C}(\alpha)$ denotes the set of clauses of $\alpha$;
- $\mathcal{F}(\alpha)$ denotes the set of fact clauses of $\alpha$;
- $\mathcal{G}(\alpha)$ denotes the set of goal clauses of $\alpha$;
- $\mathcal{R}(\alpha) := \mathcal{C}(\alpha) \setminus (\mathcal{F}(\alpha) \cup \mathcal{G}(\alpha))$ denotes the set of rule clauses of $\alpha$;

$\alpha$ is a Horn formula iff its clauses are Horn clauses.

Definition 8 (Canonical Net Representation) Let $\alpha$ be a CNF-formula and let $\mathcal{N}_\alpha = (S_\alpha, T_\alpha, F_\alpha)$ be a p/t-net; $\mathcal{N}_\alpha$ is the canonical p/t-net representation of $\alpha$ iff

- $S_\alpha = \mathcal{A}(\alpha)$ (set of atoms of $\alpha$) and $T_\alpha = \mathcal{C}(\alpha)$ (set of clauses of $\alpha$);
- for all $\tau = \neg a_1 \lor \cdots \lor \neg a_m \lor b_1 \lor \cdots \lor b_n \in \mathcal{C}(\alpha)$, where $\{a_1, \ldots, a_m, b_1, \ldots, b_n\} \subseteq \mathcal{A}(\alpha)$, $F_\alpha$ is determined by

$\tau = \{a_1, \ldots, a_m\}$, $\tau = \{b_1, \ldots, b_n\}$, i.e. the atoms $a_1, \ldots, a_m$ which are negated in the clause $\tau$ are the input places, the non-negated atoms $b_1, \ldots, b_n$ are the output places of the transition $\tau$.

The transition $\tau$ is called fact (goal, rule) transition iff the clause $\tau$ is a fact (goal, rule) clause.

Remark 1

In non-canonical p/t-net representations, $S_\alpha$ contains negated atoms (see [7]).
representations), acde is a logical consequence of the other clauses. For example $I_7$:

$$
\begin{align*}
\gamma_7 &= t_{18} = \neg acde \\
\varepsilon_7 &= \{t_{16}, t_{14}\} = \{lo, igir\} = lo \land igir \\
\varrho_7 &= t_8 = \neg lo \lor \neg igir \lor acde = lo \land igir \rightarrow acde
\end{align*}
$$

so, $\neg \gamma_7 = acde$ is a logical consequence of $\varepsilon_7 \land \varrho_7$.

Moreover, $\varepsilon_7 \land \varrho_7$ is not contradictory since in its canonical net representation $t_{18}$ is missing such that the empty marking 0 is not reproducible (see theorem 1). So $\varepsilon_7$ is an explanation of $\neg \gamma_7 = acde$.

Alltogether,

$$
\begin{align*}
\varepsilon_5 &= \{lo, igno\} = lo \land igno \\
\varepsilon_6 &= \{nolo, igno\} = nolo \land igno \\
\varepsilon_7 &= \{lo, igir\} = lo \land igir \\
\varepsilon_8 &= \{nolo, igir\} = nolo \land igir
\end{align*}
$$

are the explanations of acde.

Definition 10 Let $\alpha$ be a Horn formula and $P_\alpha : \mathbb{C}(\alpha) \rightarrow [0, 1]$ a real function, called a probability function of $\alpha$; let $H \subseteq \mathbb{F}(\alpha)$ be a set of fact clauses; let $\{D_1, \ldots, D_n\}$ be a partition of $H$ (i.e. $D_i \cap D_j = \emptyset$ for $i \neq j, \bigcup_{i=1}^n D_i = H$) where for all $D_i, 1 \leq i \leq n, \sum_{\varphi \in D_i} P_\alpha(\varphi) = 1$;

then the sets $D_1, \ldots, D_n$ are called disjoint classes;

let be $P_\alpha(\gamma) := 1$ for all goal clauses $\gamma \in G(\alpha)$, let be $E \subseteq H, R \subseteq \mathbb{R}(\alpha) \cup \mathbb{F}(\alpha), \gamma \in G(\alpha)$ and let $\varepsilon = \bigwedge_{\varphi \in E} \psi$, $\varrho = \bigwedge_{\alpha \in R} \kappa$ be the corresponding Horn formulas, where $\varepsilon$ is an explanation (diagnosis) of $\gamma$.

The probability of $\varepsilon$ is given by $P_\alpha(\varepsilon \land \varrho)$. The problem to find explanations is the probabilistic Horn abduction (PHA).

Let furthermore $I$ be a $t$-invariant of the canonical net representation $N_\alpha$ of $\alpha$ such that $I$ performs the 0-reproduction, induced by $\varepsilon \land \varrho \land \gamma$ being contradictory; then $\prod_{I \in I \setminus \{I_7\}} P_\alpha(I)$ equals the probabilities of $\varepsilon$ and of $\neg \gamma$ w.r.t. $I$.

Remark 3
The atoms of $\alpha$ are now to be interpreted as random variables.

The atoms of the fact clauses in a disjoint class $D$ form together with $P_\alpha$ a finite probability space.

Remark 4
For interpreting the probability function $P_\alpha$, let $\tau = \neg a_1 \lor \cdots \lor \neg a_m \lor b$ be a Horn clause of $\alpha$ where $\neg A = \{\neg a_1, \ldots, \neg a_m\}$, $B = \{b\}$:

- if $\tau$ is a fact clause ($\tau \in \mathbb{F}(\alpha)$, $\neg A = \emptyset$, $B \neq \emptyset$), $P_\alpha(\tau)$ is the prior probability $P(b)$ of $b$. 

![Fig. 1. $N_\alpha$ of Example 1](image-url)
• if $\tau$ is a rule clause ($\tau \in \Re(\alpha), \neg A \neq \emptyset, B \neq \emptyset$), $P_\alpha(\tau)$ is the conditional probability $P(b \mid a_1, \ldots, a_m)$ of $b$ given $a_1, \ldots, a_m$.

• if $\tau$ is a goal clause ($\tau \in \mathcal{G}(\alpha), \neg A \neq \emptyset, B = \emptyset$), the value of $P_\alpha(\tau)$ is not relevant for any calculation; from a logical point of view, the value 0 is justified because every 0-reproduction is an indirect proof and results in a contradiction; the value 1 (see Definition 10) is a very handy compromise.

**Example 3 (see Examples 1 and 2)**

The probability function $P_\alpha$ with two disjoint classes is shown in Table III. We want to calculate the probabilities of $\text{acde}$ and its explanations. There are four t-invariants passing through $t_{18} = \neg \text{acde}$: $\{I_i \mid 1 \leq i \leq 12, I_i(t_{18}) \neq 0\} = \{I_5, I_6, I_7, I_8\}$ (see Table II). The explanations of $\text{acde}$ are $\varepsilon_i = \|I_i\| \cap \mathbb{F}(\alpha)$ for $5 \leq i \leq 8$:

\[
\begin{align*}
\varepsilon_5 &= \{\text{lo}, \text{igno}\} = \text{lo} \land \text{igno} \\
\varepsilon_6 &= \{\text{nolo}, \text{igno}\} = \text{nolo} \land \text{igno} \\
\varepsilon_7 &= \{\text{lo}, \text{igir}\} = \text{lo} \land \text{igir} \\
\varepsilon_8 &= \{\text{nolo}, \text{igir}\} = \text{nolo} \land \text{igir}
\end{align*}
\]

In simple cases like this one, or if it is not necessary to watch the simulation of the (net representation of the) t-invariants, we calculate immediately:

\[
P(\varepsilon_i) = \prod_{t \in \|I_i\|} P_\alpha(t)
\]

(Please note that for the goal transitions $P_\alpha(t) = 1.0$ holds.)

\[
\begin{align*}
P(\varepsilon_5) &= 0.9 \cdot 0.4 \cdot 0.8 \cdot 1.0 = 0.288 \text{ (max.)} \\
P(\varepsilon_6) &= 0.9 \cdot 0.6 \cdot 0.0 \cdot 1.0 = 0.0 \\
P(\varepsilon_7) &= 0.1 \cdot 0.4 \cdot 1.0 \cdot 1.0 = 0.04 \\
P(\varepsilon_8) &= 0.1 \cdot 0.6 \cdot 0.6 \cdot 1.0 = 0.036
\end{align*}
\]

$P(\text{acde})$ sums up to 0.364. In case of simulating the four t-invariants, transition $t_{18}$ (acceleration delayed) would fire for $ad = 0.288, 0.04,$ and 0.036.

In order to combine the probability aspects with the propagation abilities of PNs, we introduce a new class of nets in two steps. Fig. 2(a) shows the net representation of t-invariant $I_5$. For the calculation of $P(\varepsilon_5)$ it would be convenient to have the following sequence of markings:

1) $M$ with $M(\text{lo}) = M(\text{igno}) = M(\text{acde}) = \emptyset$ (empty marking)

2) $M'(\text{lo}) = (P(\text{lo})) = (P_\alpha(t_{16})) = (0.4)$;

3) $M''(\text{lo}) = (P(\text{lo}) \cdot P(\text{igno}) \cdot P(\text{acde} \mid \text{lo} \land \text{igno})) = (0.4 \cdot 0.9 \cdot P_\alpha(t_{17})) = (0.4 \cdot 0.9 \cdot 0.8) = (0.288)$ after one subsequent firing of $t_{17}$

4) $M''' = M$ (empty marking) after one subsequent firing of $t_{18}$.

To get all that in accordance with the notation of a suitable higher level PN (predicate/transition net notation in this case) we have to complete the net as shown in Fig. 2(b).

**Definition 11** (Probability Propagation Net, PPN) Let $\alpha$ be a Horn formula and $\tau = \neg a_1 \lor \cdots \lor \neg a_m \lor b$ a Horn clause of $\alpha$ with $\neg A = \{\neg a_1, \ldots, \neg a_m\}, B = \{b\}$; $\mathcal{PN}_\alpha = (\mathcal{S}_\alpha, T_\alpha, F_\alpha, P_\alpha, L_\alpha)$ is a probability propagation net (PPN) for $\alpha$ iff:

- $\mathcal{N}_\alpha = (\mathcal{S}_\alpha, T_\alpha, F_\alpha)$ is the canonical net representation of $\alpha$.
- $P_\alpha$ is a probability function for $\alpha$.
- $L_\alpha$ is an arc label function for $\alpha$ where for $\tau$ the following holds:
  - if $\tau$ is a fact clause ($\tau \in \mathbb{F}(\alpha), \neg A = \emptyset, B \neq \emptyset$), $L_\alpha(\tau, b) = (P_\alpha(\tau))$, $(\tau, b) \in F_\alpha$ (see Fig. 3(a))
  - if $\tau$ is a rule clause ($\tau \in \Re(\alpha), \neg A \neq \emptyset, B = \emptyset$), $L_\alpha(\tau, b) = (\xi_i)$ for $1 \leq i \leq m$ $L_\alpha(\tau, b) = (\xi_1 \cdots \xi_m \cdot P_\alpha(\tau))$
where the $\xi_i$ are variables ranging over $[0, 1]$ (see Fig. 3(b))
- if $\tau$ is a goal clause ($\tau \in \mathbb{G}(\alpha), \neg A \neq \emptyset, B = \emptyset$)
  \( L_\alpha(a_i, \tau) = (\xi_i) \) for $1 \leq i \leq m$ (see Fig. 3(c)).

**Definition 12 (PPN Marking)** Let $\alpha$ be a Horn formula and $\mathcal{PN}_\alpha = (S_\alpha, T_\alpha, F_\alpha, P_\alpha, L_\alpha)$ a PPN for $\alpha$; let $W$ be a finite subset of $[0, 1]$, and let $(W) := \{(w) \mid w \in W\}$ be the corresponding set of $1$-tuples; let be $\tau \in T_\alpha$ with $\tau = \{s_1, \ldots, s_m\}$, $\tau = \{s_{m+1}\}$ (i.e. $\tau = \neg s_1 \lor \cdots \lor \neg s_m \lor \neg s_{m+1}$); then $M : S_\alpha \rightarrow M(W)$ is a marking of $\mathcal{PN}_\alpha$.

- $\tau$ is enabled by $M$ for $\{(w_1), \ldots, (w_m)\}$ iff $(w_1) \in M(s_1), \ldots, (w_m) \in M(s_m)$.
- the follower marking $M'$ of $M$ after one firing of $\tau$ for $\{(w_1), \ldots, (w_m)\}$ is given by
  \[
  M'(s_1) = M(s_1) - (w_1),
  
  \vdots
  
  M'(s_{m+1}) = M(s_{m+1}) - (w_m \cdot w_2 \cdot \ldots \cdot w_m \cdot P_\alpha(\tau));
  
  if $(\xi_1), \ldots, (\xi_m)$ are the arc labels of $(s_1, \tau), \ldots, (s_m, \tau) \in F_\alpha$, we may write
  \[
  M'(s_1) = M(s_1) - (\xi_1), \ldots, M'(s_m) = M(s_m) - (\xi_m),
  
  M'(s_{m+1}) = M(s_{m+1}) + (\xi_1 \cdot \ldots \cdot \xi_m \cdot P_\alpha(\tau)),
  
  if the $\xi_i$ are bound by the corresponding $w_i, 1 \leq i \leq m$.

**Example 4 (see Example 3)**
The net $\mathcal{PN}_\alpha$ of Fig. 4 is the PPN that combines the net $\mathcal{N}_\alpha$ of Fig. 1 and the probabilities of Table III.

The probabilities of Example 3 will now be calculated by simulating the t-invariants $I_5, I_6, I_7, I_8$. For example, simulating $I_5$ yields the maximal probability $P(\xi_5) = 0.288$. Firing $t_{13}$ and $t_{16}$ yields tuples $(0.9)$ and $(0.4)$ on places igno and lo, respectively. Firing $t_7$ takes away these tuples and puts the tuple $(0.4 \cdot 0.9 \cdot 0.8) = (0.288)$ on place acde, from where it is taken away by $t_{18}$ — such completing the reproduction of the empty marking by simulating $I_5$.

A major problem, the "loopiness", arises from the fact that the conjunction operator $\land$ is idempotent ($a \land a = a$), but the corresponding product of probabilities is not idempotent in general:

\[
P(a) \cdot P(a) \begin{cases} = P(a) & \text{if } P(a) = 1 \text{ or } P(a) = 0 \\ \neq P(a) & \text{else} \end{cases}
\]

The following example shows a case of loopiness and a method to get over that difficulty.

**Example 5 (see Example 4)**
We want to calculate the probability of $acde \land igno$. For that, we modify the PPN of Fig. 4 in several steps:
- transitions (goal clauses) $t_{18} = \neg acde$ and $t_{20} = \neg igno$ are unified to one transition (goal clause) $t_{20} = \neg acde \lor \neg igno = \neg (acde \land igno)$;
- the transitions $t_{19} = \neg owof$ and $t_{17} = \neg acno$ are omitted because they are not needed any more; as a consequence, also $t_9, t_{10}, t_1, t_2, t_5, t_6$ are no longer needed.
- all t-invariants with transitions $t$, where $P_\alpha(t) = 0$, are omitted: $t_{11}, t_{13}$;
- $t_{15}$ and $t_4$ are omitted because the only t-invariant they belong to contains a factual contradiction: $t_{16}$ (lack of oil) and $t_{15}$ (no lack of oil). The result is the PPN $\mathcal{PN}_\alpha'$ shown in Fig. 5. From a structural point of view, this net is well suited for solving our problem because its set of t-invariants (see Table IV) is reduced to the relevant ones. From a probabilistic point of view, we first of all have to note that the net is loopy. On the other hand, the net is optimal to apply Pearl's conditioning method [4]. In contrast to his technique to cut the loops, we do not need to cut the net because of the t-invariant structure that forces to fire $t_{16}$ twice in both t-invariants (see Table IV). This, in principle, leads to a double effect of $I_5$ when $t_{20}$ fires (via igno and via acde). For $L_\alpha(t_{16}, lo) = (P(t_{16})) = (1.0)$, however, this effect is neutralized. So, by simulating or simply multiplying the probabilities, we get for the t-invariants the following temporary values:

- $I_1 : P_\alpha(t_{16}^2) \cdot P_\alpha(t_{14}) \cdot P_\alpha(t_{12}) \cdot P_\alpha(t_5) \cdot P_\alpha(t_6) = 0.1$
- $I_2 : P_\alpha(t_{16}^2) \cdot P_\alpha(t_{13}) \cdot P_\alpha(t_{12}) \cdot P_\alpha(t_7) \cdot P_\alpha(t_{20}) = 0.72$

Finally, both values have to be multiplied by the weight 0.4 which is the original value of $P_\alpha(t_{16})$:

\[
P(acde \land igno) = 0.04 \quad \text{w.r.t. } I_1
\]

\[
P(acde \land igno) = 0.288 \quad \text{w.r.t. } I_2
\]
These values are also the probabilities for the two explanations:

$$\varepsilon_1 : \{lo, \text{igir}\} = lo \land \text{igir}, P(\varepsilon_1) = 0.04$$
$$\varepsilon_2 : \{lo, \text{igno}\} = lo \land \text{igno}, P(\varepsilon_2) = 0.288.$$  

Finally, $P(\text{acde} \land \text{owon}) = 0.04 + 0.288 = 0.328.$

For the representation of BNs, foldings of PPNs are appropriate. Since we do not need the formal definition, we will be content with an example.

The higher PPN $\mathcal{FPN}_\alpha$ shown in Fig. 6 is a folding of the PPN $\mathcal{PN}_\alpha$ depicted in Fig. 4. The variable mapping of the folding is shown in Table V. Let’s assume that the initial marking $M_0$ is the empty marking. After firing of $t_{lo}$ and $t_{ii}$, the marking changed into $M_1$ with

$$M_1(p) = \begin{cases} (0.4, 0.6) & \text{if } p = \text{lo} \\ (0.1, 0.9) & \text{if } p = \text{ii} \\ \emptyset & \text{else} \end{cases}$$

If $t_{\text{r}_1}$ fires $\text{lo}$ and $\text{ii}$ are cleared and $ad = (\text{lo} \times \text{ii}) \cdot \begin{pmatrix} 0.4 & 0.0 \\ 0.0 & 0.6 \\ 0.0 & 0.4 \\ 0.0 & 0.1 \end{pmatrix}$ is put on $\text{acde}$:

$$(\text{lo} \times \text{ii}) = ((0.4, 0.6) \times (0.1, 0.9)) = (0.04, 0.36, 0.06, 0.54)$$

$$ad = (0.04, 0.36, 0.06, 0.54) \cdot \begin{pmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.8 & 0.2 & 0.0 \\ 0.0 & 0.4 & 0.0 & 1.0 \end{pmatrix}$$

$$= (0.364, 0.636) \quad \text{(see Example 3).}$$

As it is common use in PN theory that foldings of nets of a certain net class are called higher nets, foldings of PPNs are called higher PPNs.
IV. BAYESIAN NETWORKS AND HIGHER PROBABILITY PROPAGATION NETS

In this section, we will show how BNs can be represented by higher PPNs. It will turn out that the structure of BNs is a bit meager for modeling directed flows of values (probabilities and likelihoods). Likelihoods are conditional probabilities in a certain interpretation. Let \( S \) be a symptom (manifestation) and \( D \) be a diagnosis (hypothesis). Then \( P(D | S) \) is a "diagnostic" probability, \( P(S | D) \) is a "causal" probability. Bayes’ rule combines both probabilities:

\[
P(D | S) = \frac{P(S | D) \cdot P(D)}{P(S)}
\]

In case of several conceivable diagnoses \( D_i, 1 \leq i \leq n \), \( P(S | D_i) \) is a measure of how probable it is that \( D_i \) causes \( S \). So, \( P(S | D_i) \) is a degree of confirmation that \( D_i \) is the cause for \( S \) which is called the "likelihood of \( D_i \) given \( S \)."

**Definition 13 (Bayesian Network)** Let \( \mathcal{B} = (\mathcal{R}, \mathcal{E}) \) be a directed acyclic graph with the set \( \mathcal{R} \) of nodes and the set \( \mathcal{E} \) of edges; let for every \( r \in \mathcal{R} \) \( \text{par}(r) \) be the set of parent nodes of \( r \):

\( \mathcal{B} \) is a Bayesian Network (BN) iff \( \mathcal{R} \) equals a set of random variables and to every \( r \in \mathcal{R} \) the table \( P(r | \text{par}(r)) \) of conditional probabilities is assigned. \( P(r | \text{par}(r)) \) indicates the prior probabilities of \( r \) if \( \text{par}(r) = \emptyset \).

**Definition 14** Let \( A = (a_1, \ldots, a_n), B = (b_1, \ldots, b_n) \) be non-negative real vectors:

\[
A \circ B := (a_1 \cdot b_1, a_2 \cdot b_2, \ldots, a_n \cdot b_n)
\]

\[
A \times B := (a_1 \cdot b_1, \ldots, a_1 \cdot b_n, a_2 \cdot b_1, \ldots, a_n \cdot b_1, \ldots, a_n \cdot b_n)
\]

We will introduce the Petri net representation of BNs by means of examples. The Petri nets are absolutely transparent and reveal the respective situation of algorithms and belief propagation (see [4], [9]).

The following example is a shortened version of the scenario of Example 6.

**Example 7**

The directed acyclic graph together with the probabilities assigned to the nodes in Fig. 7 is a BN \( \mathcal{B} \). Furthermore, it is noted that messages \( \pi \) (probabilities) and \( \lambda \) (likelihoods) flow in both directions via the edges from node to node.

Fig. 8 shows the Petri net representation \( \mathcal{PB} \) of the BN \( \mathcal{B} \). In order to initialize the net, the transitions \( \pi_{lo} \) and \( \pi_{ii} \) fire, thus putting the tuples \((0.4, 0.6)\) and \((0.1, 0.9)\) on places \( lo \) and \( ii \), respectively. \( P(lo) = 0.4, P(\neg lo) = 0.6, P(ii) = 0.1, P(\neg ii) = 0.9 \) reflect our current feeling about the possibility that lack of oil or irregular ignition happens. Based on these values, the probability of delayed acceleration can be calculated (exactly as in Example 6) by firing of transition.
The functions belonging to the respective transitions are shown in Table VI. In the conditional probability table $P(ad \mid lo, ii)$, $ad$ is a function of $lo$ and $ii$. $P^{ii=lo,ad}(ad \mid lo, ii)$ is the re-sorted table $P(ad \mid lo, ii)$ such that now $ii$ is written as a function of $lo$ and $ad$. In all tables 0.8 for example is the value for $ad = 1, lo = 1, ii = 0$.

To complete the initialization, by firing of $\pi_{lo}, \lambda_{ad}, f^2_{ad}$ we get

$$
\lambda(ii) = \lambda_{ad}(ii) = (\pi_{ad}(lo) \times \lambda(ad)) \cdot \begin{pmatrix} 1.0 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.2 & 0.0 \end{pmatrix} \\
= ((0.4, 0.6) \times (1.0, 1.0)) \cdot \begin{pmatrix} 1.0 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.2 & 0.0 \\ 0.4 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.2 & 0.0 \end{pmatrix} \\
= (1.0, 1.0).
$$

Similarly:

$$
\lambda(lo) = \lambda_{ad}(lo) = (\pi_{ad}(ii) \times \lambda(ad)) \cdot \begin{pmatrix} 1.0 & 0.6 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.2 & 0.0 \\ 0.2 & 1.0 & 0.0 & 0.0 \end{pmatrix} \\
= ((0.1, 0.9) \times (1.0, 1.0)) \cdot \begin{pmatrix} 1.0 & 0.6 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.2 & 0.0 \\ 0.2 & 1.0 & 0.0 & 0.0 \end{pmatrix} \\
= (1.0, 1.0).
$$

The likelihood $\lambda(ad) = (1.0, 1.0)$ and, as a consequence, $\lambda(lo) = \lambda(ii) = (1.0, 1.0)$ indicate that there is no reason or evidence to re-asses $\pi(ad), \pi(lo), \pi(ii)$. So, our initial beliefs are

$$
BEL(lo) := \alpha(\lambda(lo) \circ \pi(lo)) = \pi(lo) = (0.4, 0.6)
$$

$$
BEL(ii) := \alpha(\lambda(ii) \circ \pi(ii)) = \pi(ii) = (0.1, 0.9)
$$

$$
BEL(ad) := \alpha(\lambda(ad) \circ \pi(ad)) = \pi(ad) = (0.364, 0.636).
$$

Next, we assume the acceleration to be really delayed as a new evidence. So, we set $\lambda(ad) = (1.0, 0.0)$ which results in

$$
BEL(ad) = \alpha(\lambda(ad) \circ \pi(ad))
= \alpha((1.0, 0.0) \circ (0.364, 0.636))
= \alpha(0.364, 0.0) = (1.0, 0.0)
$$

with the normalizing constant $\alpha$. As further consequences, the beliefs of $lo$ and $ii$ change. Firing of $\pi(lo), \lambda(ad), f^2_{ad}$ leads to

$$
\lambda(ii) = \lambda_{ad}(ii) = (\pi_{ad}(lo) \times \lambda(ad)) \cdot \begin{pmatrix} 1.0 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.2 & 0.0 \\ 0.4 & 1.0 & 0.0 & 0.0 \end{pmatrix} \\
= ((0.4, 0.6) \times (1.0, 0.0)) \cdot \begin{pmatrix} 1.0 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.2 & 0.0 \\ 0.4 & 1.0 & 0.0 & 0.0 \end{pmatrix} \\
= (0.76, 0.32).
$$

Firing of $\pi(ii), \lambda(ad), f^3_{ad}$ leads to

$$
\lambda(lo) = \lambda_{ad}(lo) = (\pi(ii) \times \lambda(ad)) \cdot \begin{pmatrix} 1.0 & 0.6 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.2 & 0.0 \\ 0.2 & 1.0 & 0.0 & 0.0 \end{pmatrix} \\
= ((0.1, 0.9) \times (1.0, 0.0)) \cdot \begin{pmatrix} 1.0 & 0.6 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.2 & 0.0 \\ 0.2 & 1.0 & 0.0 & 0.0 \end{pmatrix} \\
= (0.82, 0.06).
$$

Then the beliefs of $lo$ and $ii$ are

$$
BEL(lo) = \alpha(\lambda(lo) \circ \pi(lo))
= \alpha((0.82, 0.06) \circ (0.4, 0.6))
= \alpha(0.328, 0.036) = (0.845, 0.155)
$$

$$
BEL(ii) = \alpha(\lambda(ii) \circ \pi(ii))
= \alpha((0.76, 0.32) \circ (0.1, 0.9))
= \alpha(0.076, 0.288) = (0.209, 0.791)
$$

In contrast to the initial beliefs, we now strongly believe in a lack of oil (0.845 > 0.4) and a little less in a normal ignition (0.791 < 0.9).

Lastly, we assume (after an inspection) that there is definitely no lack of oil. So, in addition to $\lambda(ad) = (1.0, 0.0)$ we set $\pi(lo) = (0.0, 1.0)$. Thus, the new belief of $lo$ is $BEL(lo) = (0.0, 1.0)$, and the belief of $ii$ changes:

$$
\lambda(ii) = \lambda_{ad}(ii) = (\pi(lo) \times \lambda(ad)) \cdot \begin{pmatrix} 1.0 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.2 & 0.0 \\ 0.4 & 1.0 & 0.0 & 0.0 \end{pmatrix} \\
= ((0.0, 1.0) \times (1.0, 0.0)) \cdot \begin{pmatrix} 1.0 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.2 & 0.0 \\ 0.4 & 1.0 & 0.0 & 0.0 \end{pmatrix} \\
= (0.6, 0.0).
$$

$$
BEL(ii) = \alpha(\lambda(ii) \circ \pi(ii)) = \alpha((0.6, 0.0) \circ (0.1, 0.9))
= \alpha(0.06, 0.0) = (1.0, 0.0).
$$

If lack of oil is not the reason for the delayed acceleration, we have to believe in an irregular ignition.

In Example 2 we found four explanations of a delayed acceleration:

$$
\varepsilon_5 = lo \land \neg ii,
\varepsilon_6 = \neg lo \land \neg ii,
\varepsilon_7 = lo \land ii,
\varepsilon_8 = \neg lo \land ii.
$$

$\varepsilon_5$ is the most probable explanation with $P(\varepsilon_5) = 0.288$. Since the numbers of explanations (like the t-invariants) might grow exponentially, the calculation of all explanations and their probabilities and then looking for the most probable one is obviously no reasonable approach. Much better is a modification of the Petri net approach shown in Example 8. Instead of using the usual matrix product $(A \cdot B)_{ik} = \sum_{j=1}^{n} a_{ij} \cdot b_{jk}$ (sum-product rule), we will use $(A \cdot B)_{ik} = \max\{a_{ij} \cdot b_{jk} \mid j = 1, \ldots, n\}$ (max-product rule; see [4], [9]).

Example 8 (see Example 7)

We start off with the assumption that the acceleration is delayed, i.e. $\lambda(ad) = (1.0, 0.0)$ and $BEL(ad) = (1.0, 0.0)$. Firing of $\pi_{lo}$ and $\lambda_{ad}$ puts the tuples (0.4, 0.6) and (1.0, 0.0) on places $lo$ and $ad$, respectively. Firing of $f^2_{ad}$ takes them away and puts the following tuple on place $ii$:

$$
\lambda(ii) = \lambda_{ad}(ii) = (\pi(lo) \times \lambda(ad)) \cdot \begin{pmatrix} 1.0 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.2 & 0.0 \\ 0.4 & 1.0 & 0.0 & 0.0 \end{pmatrix} \\
= ((0.4, 0.6) \times (1.0, 0.0)) \cdot \begin{pmatrix} 1.0 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.2 & 0.0 \\ 0.4 & 1.0 & 0.0 & 0.0 \end{pmatrix} \\
= (\max\{0.4, 0.0, 0.36\}, \max\{0.32, 0.0\})
= (0.4, 0.32).
$$
Correspondingly, firing of \( \pi_{ii}, \lambda, f_{ad} \) leads to the following tuple on place \( lo \):

\[
\lambda(lo) = \lambda_{ad}(lo) = (\pi(ii) \times \lambda(ad)) \bullet \begin{pmatrix} 1.0 & 0.6 \\ 0.0 & 0.4 \\ 0.2 & 1.0 \end{pmatrix} \\
= ((0.1, 0.9) \times (1.0, 0.0)) \bullet \begin{pmatrix} 1.0 & 0.6 \\ 0.0 & 0.4 \\ 0.2 & 1.0 \end{pmatrix} \\
= (\max\{0.1, 0.0, 0.72\}, \max\{0.06, 0.0\}) \\
= (0.72, 0.06).
\]

The corresponding beliefs are:

\[
BEL(lo) = \alpha(\lambda(lo) \circ \pi(lo)) \\
= \alpha((0.72, 0.06) \circ (0.4, 0.6)) \\
= \alpha(0.288, 0.036) \\
= (0.880, 0.111)
\]

\[
BEL(ii) = \alpha(\lambda(ii) \circ \pi(ii)) \\
= \alpha((0.4, 0.32) \circ (0.1, 0.9)) \\
= \alpha(0.04, 0.288) \\
= (0.122, 0.878)
\]

The most probable explanation given \( BEL(ad) = (1.0, 0.0) \) is \( lo \land \neg ii \) (or \( lo \land \neg ignored \) in the notation of Example 2).

\[\square\]

V. TRANSLATING BAYESIAN NETWORKS INTO HIGHER PROBABILITY PROPAGATION NETS

In this section, we will continue with representing BNs by PNs. In the previous examples we have shown the way higher PPNs work and that they generate the proper values. The next examples are to show that higher PPNs also satisfy the propagation formulas of BNs. We will show that only by means of these examples because the equivalence of the propagation in both approaches is easy to recognize. A formal proof would not be harder, but would also not facilitate the understanding of the equivalence.

We would like to point out that the explicit concept of a situation (marking) in PPNs is a helpful completion to the implicit concept of a situation in BNs.

In essence, there are two structural elements in BNs which are shown in Fig. 9. Fig. 9(a) indicates the probabilistic dependence of \( Y \) given \( X_1, \ldots, X_n \). So, the probability of \( Y \) is defined as \( P(Y \mid X_1, \ldots, X_n) \). Case \( n = 1 \) can be found in Example 9, cases \( n = 1 \) and \( n = 2 \) in Example 10.

There are transitions in the PPNs which are closely related to these conditional probabilities, \( f^1 \sim P(Y \mid X_1, \ldots, X_n) \). The superscript 1 points to a conditional probability. Missing superscripts denote prior probabilities. Superscripts \( \geq 2 \) point to (generalized) transposes of the probability tables (see Tables VIII and XI). For example, \( f^2_A \) is the transition that belongs to the transpose of \( (f^1_A \sim P(A \mid BC)) \) where \( C \) is written as depending on \( A \) and \( B \), in symbols \( P_{BC-AB}(A \mid BC) \) (e.g. the value belonging to \( A = 2, B = 1, C = 2 \) is equal to 0.1 in both tables). For \( n = 1 \) the transposes coincide with the normal transpose of a matrix (see Table XI). Example 10 is to shed light on this substructure and its PN representation.

The transitions in the PPNs representing structure elements according to Fig. 9(b) cause a component-wise multiplication of vectors with equal length. Let the vectors representing \( X_1, X_2, \ldots, X_n \) have length \( m \); then the transition \( \lambda_X \) transforms the vectors \( \lambda_X(Y_1), \ldots, \lambda_X(Y_n) \) into

\[
\lambda(X) = \lambda_X(Y_1) \circ \cdots \circ \lambda_X(Y_n)
\]

(see Definition 14). \( m_X^1 \) is the only transition that causes a product of only \( \lambda \)-factors. Superscripts \( \geq 2 \) belong to mixtures of \( \lambda \)- and \( \pi \)-factors. Example 9 is to throw light on this structural element (with \( n = m = 2 \)) and its PN representation.

The next two examples are borrowed from [9] as well as the definitions, propagation formulas and statements we will apply in the sequel. They all are collected in the appendix.

Example 9 (Cheating spouse)
The scenario consists of a spouse and a strange man/lady. It has to be reported that spouse might be cheating. As a consequence, there are four important random variables: spouse is cheating \( A \), spouse dines with another \( B \), spouse is reported seen dining with another \( C \), strange man/lady calls on the phone \( D \).

The BN with prior and conditional probabilities is shown in Fig. 10. The random variables \( A, B, C, D \) have two attributes \((\neg - \text{ and } \neg \text{ meaning "yes" and "no"})\) which are listed in Table VII.

The functions belonging to the respective transitions are shown in Table VIII. As stated above, the structure of BNs is a bit meager and the actual steps of probability propagation are "hidden" in the algorithms. In contrast to that, the PN representation detailedly shows the probability propagation, and the algorithms are distributed over the net such that each transition’s share is of manageable size. In spite of the
TABLE VIII
TRANSITION FUNCTIONS OF EXAMPLE 9

\[\begin{align*}
f_A &\simeq P(A) = \\
f_B &\simeq P(B | A) = \\
f_C &\simeq P(C | B) = \\
f_D &\simeq P(D | A) = \\
f_B &\simeq P(A-B | A) = \\
f_C &\simeq P(B-C | C) = \\
f_D &\simeq P(A-D | D) =
\end{align*}\]

![Diagram of Example 9](image)

Fig. 11. PBN of Example 9

![Diagram of λ(t)-invariant of PBN](image)

Fig. 12. λ(A)-t-invariant of PBN

The exactness of the PN representation, one might consider the size of the net a disadvantage. On the other hand, the size of the PPNs might indicate that BNs are indeed a little under-structured. Moreover, the specific structure of PPNs causes an absolutely appropriate partition into propagation processes. The minimal t-invariants (precisely their net representations) describe exactly the paths of probabilities and likelihoods. The t-invariants need not to be calculated on the higher level. It is sufficient to calculate them on the “black” net (i.e. the underlying place/transition net without arc labels).

The PN representation of the BN in Fig. 10 is shown in Fig. 11. The three minimal t-invariants are shown in Table IX, their net representations in Fig. 12–14. Due to lack of space, the rules for calculating the output tuples of the transitions are missing but they are specified in the net representations of all t-invariants (Fig. 12, 13, 14). The three minimal t-invariants in vector form are given in Table IX. The t-invariants as solutions of a homogeneous linear equation system result in net representations in which markings are reproducible. In our case, the PNs are cycle-free and have a transition boundary. This implies the reproducibility of the empty marking by every t-invariant as a flow of tuples from input to output boundary. Fortunately, these flows describe exactly the flow of λ- and π-messages. So, the net representations of t-invariants are a framework for the propagation of λ- and π-tuples. We will show that by explaining how the PNs work and refer at each step to the corresponding definitions, lemmas, and propagation formulas of [9] which we collected in the appendix.
In the initialization phase, we first set all $\lambda$-tuples to $(1,0,1,0)$ (initialization rule (1), see Appendix, section B), thus expressing that we have no external evidence concerning the random variables. Next, we set $\pi(A) = P(A) = (0.1,0.9)$ (initialization rule (2)), thus preparing the propagation of $P(a_1) = 0.1, P(a_2) = 0.9$ as a $\pi$-message. Finally, we have to calculate $P(C)$ and $P(D)$. For that we will use the $\pi(C)$- and $\pi(D)$-t-invariant (Fig. 13 and 14). In Fig. 13, the boundary transitions $\pi_A$ and $\lambda_D$ are enabled because they have no input places. When firing, $\pi_A$ puts the tuple $\pi(A) = (0.1,0.9)$ on place $A$ (initialization rule (2)), $\lambda_D$ puts $\lambda(D) = (1,0,1,0)$ on place $D$ (initialization rule (1)). Now, transition $f_B^1$ is enabled and removes $(1,0,1,0)$ from place $D$ and puts $\lambda_D(D) = \lambda(D) \cdot (0.2 \ 0.3 \ 0.8) = (1,0,1,0)$ on place $A$ (operative formulas (1), (3), see Appendix, section A). (The boundary transitions $\pi_A$ and $\lambda_D$ are permanently enabled. But for reproducing the empty marking they have to fire only once. See Table IX where their entries are 1 in the corresponding row.)

The next transition to fire is $m_A^3$. It takes $\pi(A) = (0.1,0.9)$ and $\lambda_D(A) = (1,0,1,0)$ from the respective places $A$ and puts $\pi_B(A) = \pi(A) \circ \lambda_D(A) = (0.1,0.9)$ on place $A$ (Definition 16, see Appendix, section C).

(When a tuple is removed from a place $X$, that does not mean that the values of the tuple are no longer valid for $X$. It simply says that they have been used and that they can be re-generated any time.)

Next, the transitions $f_B^1, f_C^1, \pi_C$ are enabled and fire in that sequence. By firing of $f_B^1$ the tuple $(0.1,0.9)$ is taken from $A$, and the tuple $\pi(B) = (0.1,0.9) \cdot (0.7 \ 0.3 \ 0.2) = (0.25,0.75)$ is put on $B$ (operative formula (4)). This tuple is taken away by firing of $f_D^1$, and the tuple $\pi(C) = (0.25,0.75) \cdot (0.4 \ 0.6) = (0.10075,0.89925)$ is put on $C$ (operative formula (4)) from where it is removed by transition $\pi_C$, thus completing the reproduction of the empty marking.

The probabilities $P(B)$ and $P(C)$ are calculated as follows:

$$P(B) = \alpha(\lambda(B) \circ \pi(B)) = \alpha((1,0,1,0) \circ (0.25,0.75))$$
$$= (0.25,0.75)$$

$$P(C) = \alpha(\lambda(C) \circ \pi(C))$$
$$= \alpha((1,0,1,0) \circ (0.10075,0.89925))$$
$$= (0.10075,0.89925)$$

Similarly $P(D) = (0.44,0.56)$ is calculated on the basis of the $\pi(D)$-invariant (Fig. 14).

Now, we assume that $B$ is instantiated: $\lambda(B) = (1,0,0,0)$ which means that spouse dines with another. The consequence for the PNs in Fig. 12–14 is quite simple: the variable arc labels $\lambda_C(B)$ and $\pi(B)$ are replaced by the constant tuple $(1,0,0,0)$. That means whatever the input values enabling $f_B^1$ and $f_D^1$ are, both transitions put $(1,0,0,0)$ on their respective output places $B$ (operative formulas (2), (3)).

The changes of the probabilities are as follows:
In Fig. 12:

\[ \lambda_D(A) = (1.0, 1.0) \] as before;

\[ \lambda_B(A) = \lambda(B) \cdot \left( \begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix} \right) = (1.0, 0.0) \cdot \left( \begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix} \right) = (0.7, 0.2) \] 

by firing of \( f_B^2 \);

\[ \lambda(A) = \lambda_B(A) \circ \lambda_D(A) = (0.7, 0.2) \circ (1.0, 1.0) = (0.7, 0.2) \]

\[ P(A) = \alpha(\pi(A) \circ \lambda(A)) = \alpha((0.1, 0.9) \circ (0.7, 0.2)) = (0.07, 0.18) = (0.28, 0.72) \] 

for \( \alpha = \frac{1}{0.25} \)

In Fig. 13: \( f_B^1 \) puts the tuple \((1.0,1.0)\) on place \( B \); then \( f_C^2 \) is enabled and puts

\[ \pi(C) = (1.0, 0.0) \cdot \left( \begin{bmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{bmatrix} \right) = (0.4, 0.6) \] 

on place \( C \);

\[ P(C) = \alpha(\pi(C) \circ \lambda(C)) = \alpha((1.0, 1.0) \circ (0.4, 0.6)) = (0.4, 0.6) \]

In Fig. 14: \( \lambda_B(A) = (0.7, 0.2) \) (see above). Firing of \( m_A^3 \) puts \( \pi_D(A) = \alpha(\pi(A) \circ \lambda_B(A)) = \alpha((0.1, 0.9) \circ (0.7, 0.2)) = (0.28, 0.72) \) on place \( A \); after firing of \( f_D^1 \)

\[ \pi(D) = \pi_D(A) \cdot \left( \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix} \right) = (0.28, 0.72) \cdot \left( \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix} \right) = (0.512, 0.488) \]

\[ P(D) = \alpha(\lambda(D) \circ \pi(D)) = \alpha((1.0, 1.0) \circ (0.512, 0.488)) = (0.512, 0.488) \]

The interpretation is that after spouse dines with another \( \lambda(B) = (1.0, 0.0) \) the probabilities (our beliefs) that

- spouse is cheating
- spouse is reported seen dining with another
- strange man/lady calls on the phone

are increased, now.

If we, in addition, assume that no strange man/lady calls on the phone \( \lambda(D) = (0.0, 1.0) \) the constant arc weight \((1.0, 1.0)\) at the output arc of transition \( \lambda_D \) has to be changed into \((0.0, 1.0)\). This does not change \( P(B) \) and \( P(C) \). It changes \( P(D) \) and \( P(A) \):

\[ P(D) = \alpha(\lambda(D) \circ \pi(D)) = \alpha((0.0, 1.0) \circ (0.512, 0.488)) = (0.0, 4.88) = (0.0, 1.0) \] 

for \( \alpha = \frac{1}{0.25} \)

In the \( \lambda(A) \)-t-invariant of Fig. 12 we find \( \lambda_D(A) = \lambda(D) \cdot f_B^2 = (0.0, 1.0) \cdot \left( \begin{bmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{bmatrix} \right) = (0.2, 0.6) \) on place \( A \) after firing of transition \( f_{B}^2 \). Then, after firing of transition \( m_A^3 \)

\[ \lambda(A) = \lambda_D(A) \circ \lambda_B(A) = (0.7, 0.2) \circ (0.2, 0.6) = (0.14, 0.12) \] 

was put on \( A \). So,

\[ P(A) = \alpha(\pi(A) \circ \lambda(A)) = \alpha((0.1, 0.9) \circ (0.14, 0.12)) = (0.014, 0.108) = (0.1148, 0.8852) \] 

for \( \alpha = \frac{1}{127} \).

So, the probability (our belief) that spouse is cheating is decreased.

Example 10 (Burglar Alarm)

In this equally very popular example [9], Mr. Holmes is sitting in his office when he gets a call that his burglar alarm is sounding \( (A) \). Of course, he suspects a burglary \( (B) \) (even though there might be other reasons for activating the alarm, e.g. an earthquake \( (C) \)). On his ride home, he hears on the radio an announcement about some earthquake. How do the phone call and the radio announcement influence his belief about getting burglarized?

The BN \( B \) with prior and conditional probabilities is shown in Fig. 15. The random variables \( A, B, C \) have two attributes \((-1\) and \(-2\) meaning "yes" and "no") listed in Table XI. The PN version \( \mathcal{PN} \) of \( B \) is shown in Fig. 16, the corresponding transitions functions in Table XI. Due to lack of space, the rules for calculating \( \pi(A), \lambda_A(B), \lambda_A(C) \) are missing in Fig. 16, but they are specified in the t-invariants (Fig. 17–19).

To open the initialization phase, we set all \( \lambda \)-tuples to \((1.0,1.0)\). In doing so, we state that there is no evidence to change the prior probabilities of \( B \) and \( C \) (initialization rule (1)). Moreover, we set \( \pi(B) = P(B) = (0.01,0.99) \).
TABLE XI

<table>
<thead>
<tr>
<th>Transition Functions of Example 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_B \simeq P(B) =$</td>
</tr>
</tbody>
</table>
| \[
\begin{array}{c|cc}
B & 1 & 2 \\
\hline
0.01 & 0.99 \\
\end{array}
\]
| $f_C \simeq P(C) =$ |
| \[
\begin{array}{c|cc}
C & 1 & 2 \\
\hline
0.001 & 0.999 \\
\end{array}
\]
| $f_{\lambda} \simeq P(A \mid BC) =$ |
| \[
\begin{array}{c|ccc}
A & B & C & 1 \\
\hline
1 & 1 & 0.99 & 0.01 \\
1 & 2 & 0.9 & 0.1 \\
2 & 1 & 0.5 & 0.5 \\
2 & 2 & 0.01 & 0.99 \\
\end{array}
\]
| $f_{\lambda}^2 \simeq P_A(B \rightarrow AC) =$ |
| \[
\begin{array}{c|ccc}
A & B & C & 1 \\
\hline
1 & 1 & 0.99 & 0.01 \\
1 & 2 & 0.9 & 0.1 \\
2 & 1 & 0.5 & 0.5 \\
2 & 2 & 0.01 & 0.99 \\
\end{array}
\]
| $f_{\lambda}^3 \simeq P_A(C \leftarrow AB) =$ |
| \[
\begin{array}{c|ccc}
A & B & C & 1 \\
\hline
1 & 1 & 0.99 & 0.01 \\
1 & 2 & 0.9 & 0.1 \\
2 & 1 & 0.5 & 0.5 \\
2 & 2 & 0.01 & 0.99 \\
\end{array}
\]

Fig. 17. $\pi(A)$-t-invariant of $\mathcal{PB}$

Fig. 18. $\lambda(B)$-t-invariant of $\mathcal{PB}$

Fig. 19. $\lambda(C)$-t-invariant of $\mathcal{PB}$

and $\pi(C) = P(C) = (0.001, 0.999)$ (initialization rule (2)).

Next, we calculate $\pi(A)$ by reproducing the empty marking in the $\pi(A)$-t-invariant of Fig. 17. When firing, $\pi_B$ and $\pi_C$ put tokens $(0.01, 0.99)$ and $(0.001, 0.999)$ on places $B$ and $C$, respectively. Then $f_{\lambda}^1$ is activated and fires. By that, the tuples are removed from places $B$ and $C$ and the following tuple is put on place $A$ (operative formula (4)):

$$
\pi(A) = (\pi_A(B) \times \pi_A(C)) \cdot \begin{bmatrix}
0.99 & 0.01 \\
0.9 & 0.1 \\
0.01 & 0.99
\end{bmatrix}
$$

$$(0.01, 0.99) \times (0.001, 0.999) \cdot \begin{bmatrix}
0.99 & 0.01 \\
0.9 & 0.1 \\
0.01 & 0.99
\end{bmatrix}
$$

$$
= (0.019, 0.982).
$$

So,

$$
P(A) = \alpha \cdot (\lambda(A) \circ \pi(A))
$$

$$
= \alpha ((1.0, 1.0) \circ (0.019, 0.981))
$$

$$
= (0.019, 0.982) \text{ for } \alpha = 1
$$

Firing of $\pi_B$ completes reproducing the empty marking.

Now, we assume that Mr. Holmes got the call and knows that his alarm sounds. That means $A$ has to be instantiated for $a_1$. Consequently, we have to change the arc label $(1.0, 1.0)$ of arc $(\lambda_A, A)$ to $(1.0, 0.0)$ in Fig. 16–19. In order to calculate Holmes’ present beliefs about $B$ (being burglarized) and $C$ (earthquake), we reproduce the empty marking in the $\lambda(B)$- and $\lambda(C)$-t-invariant (Fig. 18 and 19).

In Fig. 18, after firing $\lambda_A$ and $\pi_C$, tuples $(1.0, 0.0)$ and $(0.001, 0.999)$ are lying on places $A$ and $C$, respectively. Now, $f_{\lambda}^2$ is activated and fires. After that the tuples are removed...
from places $A$ and $C$ and the following tuple is put on place $B$ (operative formula (1)):

$$
\lambda_A(B) = (\lambda(A) \times \lambda_A(C)) \cdot \begin{pmatrix} 0.99 & 0.5 \\ 0.01 & 0.5 \\ 0.1 & 0.99 \end{pmatrix}
= ((1.0, 0.0) \times (0.001, 0.999)) \cdot \begin{pmatrix} 0.99 & 0.5 \\ 0.01 & 0.5 \\ 0.1 & 0.99 \end{pmatrix}
= (0.9, 0.01).
$$

Removing that tuple by $\lambda_B$ completes the reproduction of the empty marking.

In the same way, we get $\lambda_A(C) = (0.505, 0.019)$ in the $\lambda(C)$-t-invariant of Fig. 19. This leads to

$$
P(B) = \alpha \left( (\lambda(B) \circ \pi(B)) = \alpha \left( (0.9, 0.01) \circ (0.01, 0.99) \right) = \alpha(0.009, 0.0099) = (0.476, 0.524) \right. \text{ for } \alpha = \frac{1}{0.0189}
$$

$$
P(C) = \alpha \left( (\lambda(C) \circ \pi(C)) = \alpha \left( (0.505, 0.019) \circ (0.001, 0.999) \right) = \alpha(0.00505, 0.018981) = (0.026, 0.974) \right. \text{ for } \alpha = \frac{1}{0.019486}
$$

Holmes’ belief in being burglarized has increased from 0.01 to 0.476. Even his belief in an earthquake has increased from 0.001 to 0.026.

Next, we assume that Mr. Homes heard the announcement of an earthquake on the radio. Now, we have to change the arc label $(0.001, 0.99)$ of arc $(\pi_C, C)$ to $(1.0, 0.0)$ in Fig. 16–19 since $C$ has to be instantiated for $c_1$. To calculate Mr. Holmes’ belief about $B$ (being burglarized) we again reproduce the empty marking in the $\lambda(B)$-t-invariant (Fig. 18). After firing $\lambda_A$ and $\pi_C$, tuples $(1.0, 0.0)$ are lying on places $A$ and $C$, respectively. After firing $f_A^2$ these tuples are removed and the following tuple is put on place $B$:

$$
\lambda_B(B) = (\lambda(A) \times \lambda_A(C)) \cdot \begin{pmatrix} 0.99 & 0.5 \\ 0.01 & 0.5 \\ 0.1 & 0.99 \end{pmatrix}
= ((1.0, 0.0) \times (1.0, 0.0)) \cdot \begin{pmatrix} 0.99 & 0.5 \\ 0.01 & 0.5 \\ 0.1 & 0.99 \end{pmatrix}
= (1.0, 0.0, 0.0, 0.0) \cdot \begin{pmatrix} 0.99 & 0.5 \\ 0.01 & 0.5 \\ 0.1 & 0.99 \end{pmatrix}
= (0.99, 0.5).
$$

$\lambda_B$ fires and removes that tuple from $B$, thus completing the reproduction of the empty marking. Mr. Holmes’ new belief concerning $B$ is

$$
P(B) = \alpha \left( (\lambda(B) \circ \pi(B)) = \alpha \left( (0.99, 0.5) \circ (0.01, 0.99) \right) = \alpha(0.0099, 0.495) = (0.02, 0.98) \right. \text{ for } \alpha = \frac{1}{0.0049}.
$$

So, Holmes’ belief in being burglarized has changed from 0.01 via 0.476 to 0.02. Holmes was worried after the phone call and calmed down after the announcement.

VI. CONCLUSION AND FUTURE WORK

We introduced probability propagation nets (PPNs) on the basis of PN representation of propositional Horn clauses. This makes it possible to represent deduction (and abdication) processes as reproduction of the empty marking [7]. Touchstones for our approach are the representation of probabilistic Horn abduction and the propagation of $\lambda$- and $\pi$-messages in BNs.

In our opinion, it is valuable to introduce specific PN concepts into the field of propagations. In particular t-invariants as an elementary means to structure PNs turned out to be quite fruitful. The minimal t-invariants, on the one hand when reproducing the empty marking describe exactly the flows of $\lambda$- and $\pi$-messages, thus structuring the PNs (and so the BNs) in a very natural way. On the other hand, they reveal the true complexity behind the simply structured BNs.

Also the markings turned out to be useful insofar as they clearly (and completely) partition all flows in easily observable situations. Altogether, the PPNs are an additional means for describing the flows of probability and evidence that yields a lot of clarity.

In the near future, we aim at integrating PN representations of technical processes and probability propagations.

APPENDIX

A. Operative Formulas in Bayesian Networks

The following formulas used in chapter V are taken from [9].

1) If $B$ is a child of $A$, $B$ has $k$ possible values, $A$ has $m$ possible values, and $B$ has one other parent $D$, with $n$ possible values, then for $1 \leq j \leq m$ the $\lambda$ message from $B$ to $A$ is given by

$$
\lambda_B(a_j) = \sum_{p=1}^{n} \pi_B(d_p) \left( \sum_{i=1}^{k} P(b_i|a_j, d_p) \lambda(b_i) \right).
$$

2) If $B$ is a child of $A$ and $B$ has $m$ possible values, then for $1 \leq j \leq m$ the $\pi$ message from $A$ to $B$ is given by

$$
\pi_B(a_j) = \begin{cases} 1 & \text{if } A \text{ is instantiated for } a_j \\ 0 & \text{if } A \text{ is instantiated, but not for } a_j \\ \frac{P'(a_j)}{\lambda_{B}(a_j)} & \text{if } A \text{ is not instantiated,} \end{cases}
$$

where $P'(a_j)$ is defined to be the current conditional probability of $a_j$ based on the variables thus far instantiated.

3) If $B$ is a variable with $k$ possible values, $s(B)$ is the set of $B$’s children, then for $1 \leq i \leq k$ the $\lambda$ value of $B$ is given by

$$
\lambda(b_i) = \begin{cases} \prod_{C \in s(B)} \lambda_C(b_i) & \text{if } B \text{ is not instantiated} \\ 1 & \text{if } B \text{ is instantiated for } b_i \\ 0 & \text{if } B \text{ is instantiated, but not for } b_i. \end{cases}
$$
4) If $B$ is a variable with $k$ possible values and exactly two parents, $A$ and $D$, $A$ has $m$ possible values, and $D$ has $n$ possible values, then for $1 \leq i \leq k$ the $\pi$ value of $B$ is given by

$$\pi(b_i) = \sum_{j=1}^{m} \sum_{p=1}^{n} P(b_i|a_j, d_p)\pi_B(a_j)\pi_B(d_p). \tag{4}$$

5) If $B$ is a variable with $k$ possible values, then for $1 \leq i \leq k$, $P'(B_i)$, the conditional probability of $b_i$ based on the variables thus far instantiated, is given by

$$P'(b_i) = \alpha \lambda(b_i)\pi(b_i). \tag{5}$$

B. Initialization in Bayesian Networks

The following rules taken from [9] describe the initialization phase for BNs:

1) Set all $\lambda$ values, $\lambda$ messages and $\pi$ messages to 1.
2) For all roots $A$, if $A$ has $m$ possible values, then for $1 \leq j \leq m$, set $\pi(a_j) = P(a_j)$.

C. Definition of $\lambda$ and $\pi$ Messages

The following definitions are, again, taken from [9].

**Definition 15** ($\lambda$ message) Let $C = (V, E, P)$ be a causal network in which the graph is a tree, $W$ a subset of instantiated variables, $B \in V$ a variable with $k$ possible values, and $C \in s(B)$ a child of $B$ with $m$ possible values. Then for $1 \leq i \leq k$, we define

$$\lambda_C(b_i) = \sum_{j=1}^{m} P(c_j|b_i)\lambda(c_j).$$

The entire vector of values $\lambda_C(b_i)$ for $1 \leq i \leq k$, is called the $\lambda$ message from $C$ to $B$ and is denoted $\lambda_C(B)$.

**Definition 16** ($\pi$ message) Let $C = (V, E, P)$ be a causal network in which the graph is a tree, $W$ a subset of instantiated variables, $B \in V$ a variable which is not the root, and $A \in V$ the father of $B$. Suppose $A$ has $m$ possible values. Then we define for $1 \leq j \leq m$

$$\pi_B(a_j) = \begin{cases} 
1 & \text{if } A \text{ is instantiated for } a_j \\
0 & \text{if } A \text{ is instantiated, but not for } a_j \\
\pi(a_j) \prod_{C \in s(A), C \neq B} \lambda_C(a_j) & \text{if } A \text{ is not instantiated,}
\end{cases}$$

where $\lambda_C(a_j)$ is defined in definition 15. Again, if there are no terms in the product, it is meant to represent the value 1.

The entire vector of values, $\pi_B(a_j)$ for $1 \leq j \leq m$, is called the $\pi$ message from $A$ to $B$ and is denoted $\pi_B(A)$. 

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