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A Petri Net Approach for Propagating Probabilities and Mass Functions

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Abstract This paper has a twofold concern. First, to show that probability propagation nets (PPNs) can be derived from formulae for conditional probabilities. Second, to show that PPNs are usable for (local) propagations of mass distributions, too, if the inscriptions are suitably adapted.

1 Introduction

The concern of this paper is to show how probability structures and belief structures, as well as the propagations therein, can be modeled by (higher level) Petri nets (PNs). In all these fields it is common practice that the authors are very strict with regard to the "right" interpretation of their respective concepts. On the other hand, the propagations are usually not modeled with comparable detailedness. For example a directed graph as graphical structure of a Bayesian network is a bit meager to elucidate the flow of probabilities and the counter flow of evidences over one and the same directed arc. The details are hidden in the corresponding algorithms [Nea90,Pea88] whose form is absolutely unsuitable to show dynamic processes. Moreover, there is no connection between graphs and algorithms.

Also the representation of mass flows on hypertrees [SS91] is not satisfying because the actual propagations are not modeled. In [KSH91] propagations are modeled. However, the algorithms are not very convenient for representing dynamical processes.

An approach to adequately describe the propagation of probabilities and evidences can be found in [Pin07,LP05,LP07,LP11]. These papers are to present higher level Petri nets as a suitable means to transparently and exactly describe probabilistic propagations.

The probability propagation nets (PPNs) developed there are predicate/transition nets [GL79,GL83,Gen86,Mül04] which are foldings of simpler nets for describing the probabilistic Horn abduction (PHA) [Poo93].

PNs are especially qualified to model this kind of propagations because they are in line with the linear structure of the propagating algorithms. Moreover, they reveal the concurrency of the flows.

The way we introduce PPNs is peculiar in that we transform a formula (from [KSH91]) into a PN. This formula belongs to an example which is representative because there are respective nodes with input and output branching degree greater than one. By help of the same example we show that PPNs with appropriate inscriptions are also able to model the flows of masses and evidences.

The paper is organized as follows. In chapter 2 the PN structures of Horn formulae are developed (Horn PNs [Lau03]). These structures are important because we will show that all flows of probabilities and evidences uniquely belong to a Horn sub-PN of the entire net. Chapter 3 deals with dependency PNs which correspond to the dependency networks in [KSH91]. They model probability flows without evidences (initialization). Chapter 4 shows a simple technique to calculate the "generalized" transposes of conditional probability tables. Tables of the form p(c|ab) have two transposes, whereas p(b|a) has only one (the usual transpose of a matrix). In chapter 5 it is shown how to transform a formula into a PPN by changing the products of the formula to vector and matrix products. Chapter 6 introduces important concepts of the theory of mass distributions. After that it is demonstrated that the PN structure of the PPNs (including vector and matrix products) is also able to model the propagation of masses and evidences. The 7th chapter (conclusion, further work) completes the paper.

I am greatly indebted to Kerstin Susewind for her valuable assistance.

2 Petri Net Representation of Propositional Horn Formulae

In this chapter we introduce Petri Net (PN) representations of Horn formulae (formulae in CNF whose clauses are Horn clauses). The reason for that are nice properties of the nets if the Horn formulae are contradictory. In this case the empty marking is reproducible (in both directions).

Definition 1.

A literal *is an atom or the negation of an atom; a* clause *is a disjunction of literals.*

Let $t = \neg a_1 \vee \ldots \vee \neg a_m \vee b_1 \vee \ldots \vee b_n$ *be a clause; for* t *also the following set notations are in use:*

$$t = \{\neg a_1, \ldots, \neg a_m, b_1 \vee \ldots b_n\} \text{ and } t = \neg A \cup B \text{ where}$$
$$A = \{a_1, \ldots, a_m\} \ \neg A = \{\neg a_1, \ldots, \neg a_m\} \ B = \{b_1, \ldots, b_n\};$$

if $\neg A$ *is empty,* t *is a* fact clause*, if* B *is empty,* t *is a* goal clause*,*
if neither $\neg A$ *nor* B *are empty,* t *is a* rule*.*

A formula α *is in* conjunctive normal form (CNF) *iff* α *is a conjunction of clauses.* $A(\alpha)$ *denotes the* set of atoms *and* $C(\alpha)$ *denotes the* set of clauses *in a CNF-formula* α*.*

□

Definition 2.

A clause κ is a Horn clause *iff it contains at most one non-negated atom; a CNF-formula is a* Horn formula *iff its clauses are Horn clauses.*

□

Definition 3.

Let α be a Horn formula; let $N_\alpha = (S_\alpha, T_\alpha, F_\alpha)$ be the short form of a place/transition net (p/t-net) $N_\alpha = (S_\alpha, T_\alpha, F_\alpha, W_\alpha)$ *where* $W_\alpha : F_\alpha \to \{1\}$.

$N_\alpha = (S_\alpha, T_\alpha, F_\alpha)$ *is the* p/t-net representation of α *iff*

$$S_\alpha = A(\alpha),\ T_\alpha = C(\alpha),\ \text{and } F_\alpha \text{ is given by } {}^\bullet t = A_t,\ t^\bullet = B_t$$
$$\text{for all clauses } t = \neg A_t \cup B_t \in C(\alpha);$$

then N_α is a Horn PN.

□

Theorem 1. [see [Lau03] theorem 6]

Let α be a Horn formula and $N_\alpha = (S_\alpha, T_\alpha, F_\alpha)$ its p/t-net representation; then α is contradictory iff the empty marking is reproducible ($\mathbf{0}$-reproducibility).

□

Remark 1.

If N_α is **0**-reproducible by a firing sequence σ

- at least one goal transition is firable because only goal transitions empty the net,
- N_α is **0**-reproducible in backward direction by firing σ in reverse order,
- N_α has a t-invariant with a transition boundary.

□

Example 1.

$$\alpha = \underset{(1)}{a} \wedge \underset{(2)}{b} \wedge \underset{(3)}{(\neg a \vee \neg b \vee c)} \wedge \underset{(4)}{\neg c}$$

is a contradictory Horn formula.

The firing sequence (1,2,3,4) reproduces the empty marking whereby the goal transition 4 clears the net. The backward firing sequence (4,3,2,1) reproduces the empty marking, too. The whole net is the net representation of a t-invariant.

□

Notice that removing a token from a or b or c does not mean that a, b, c have become false. It rather means that the fact of a, b, c being true has been used for proving the contradiction of α.

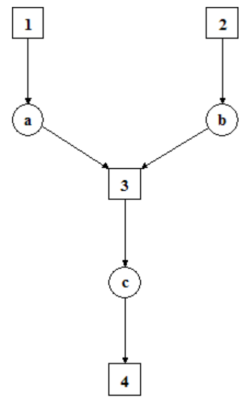

Figure 1. Horn PN

3 Dependency Petri Nets

The PNs introduced in this chapter are "overlays" of Horn PNs with transition boundaries. If a node has more than one input arc it is a transition, and if it has more than one output arc it is a place. These nets are called "dependency PNs" in order to point to dependency networks [KSH 01] which are specific hypertrees.

The dependency PNs are then upgraded to high level dependency PNs which are intended for calculating and transporting probability tuples. All minimal t-invariants are net representations of Horn PNs belonging to contradictory Horn formulae, thus guaranteeing their **0**-reproducibility.

It is important to notice now that the operative meaning of the minimal t-invariants has changed with the upgrading. All propagations of probabilities are represented by **0**-reproductions and vice versa. Actually, the goal transitions are of no importance for the calculation of probabilities. But they clear the net and complete the calculations to **0**-reproductions, thus equalizing the "rails" of probability propagations with t-invariants.

Definition 4.

Let $N = (P, T, F)$ *be a p/t-net and* $F^{-1} := \{(l, k) \mid (k, l) \in F\}$*;*

> $pa = [(k_0, k_1), (k_1, k_2), \ldots, (k_{n-1}, k_n)]$ *is an* undirected path *from* k_0 *to* k_n *in* N *iff* $\{(k_0, k_1), (k_1, k_2), \ldots, (k_{n-1}, k_n)\} \subseteq F \cup F^{-1}$,

i.e. the arrow hats are ignored or even deleted;

N *is a* dependency PN *iff*

- *for all* $k \in P \cup T : (|^\bullet k| = 0 \vee |k^\bullet| = 0) \Rightarrow k \in T$*, i.e.* N *has a transition boundary; and*

- *for all* $k \in P \cup T : (|{}^{\bullet}k| \geq 2 \;\Rightarrow\; k \in T) \wedge (|k^{\bullet}| \geq 2 \;\Rightarrow\; k \in P)$ *; and*
- *between any two nodes* $k_1, k_2 \in P \cup T$ *there is exactly one undirected path, i.e. the net is connected and circle free.*

□

Remark 2.

- $[(g,V),(V,d),(d,W),(W,h),(h,Y)]$ is an undirected path in figure 2.
- The name "dependency PN" is to remind of dependency networks [KSH 01].
- $\forall p \in P : |{}^{\bullet}p| = 1 \;\wedge\; |p^{\bullet}| \geq 1$
- $\forall t \in T : \qquad |t^{\bullet}| \leq 1$
- The net representation of every simple t-invariant in a dependency PN is a Horn PN belonging to a contradictory Horn formula α.
- Dependency PNs are loop free, because any loop $\{(p,t),(t,p)\}$ implies two undirected paths between p and t.
- Every dependency PN is covered by Horn PNs.

□

Example 2.

The dependency PN in figure 2 consists of two Horn PNs which are partly covering each other:

$$N_1 = (P_1, T_1, F_1) \text{ with } P_1 = \{a, b, d, g\} \text{ and } T_1 = \{P, Q, U, V, X\}$$
$$N_2 = (P_2, T_2, F_2) \text{ with } P_2 = \{a, b, d, h\} \text{ and } T_2 = \{P, Q, U, W, Y\}$$

The corresponding contradictory Horn formulae are

$$\alpha_1 = a \wedge b \wedge (\neg a \vee \neg b \vee d) \wedge (\neg d \vee g) \wedge \neg g$$
$$\alpha_2 = a \wedge b \wedge (\neg a \vee \neg b \vee d) \wedge (\neg d \vee h) \wedge \neg h$$

The contradictions are equivalent to the facts that

g follows from $a, b, a \wedge b \to d, d \to g$

h follows from $a, b, a \wedge b \to d, d \to h$.

The inscriptions at the transitions are the clauses the transitions stand for. They are only to again illustrate the transition's meaning.

□

Next, we want to extend dependency PNs to probability propagation nets (PPNs). In PPNs the objects "flowing" through the net are no longer tokens but probability tuples of the form

$$[a, 1-a], [a, b, 1-(a+b)] \quad \text{etc. with} \quad 0 \leq a, b, 1-a, 1-(a+b) \leq 1.$$

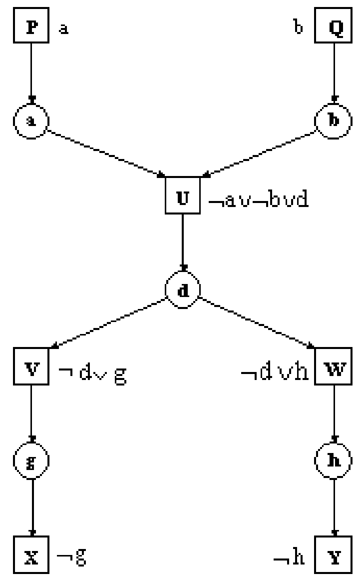

Figure 2. Dependency Petri net DN

Definition 5. [High level dependency PNs]

Let $N = (P, T, F)$ be a dependency PN and let $s \in P$ be a place;

to all arcs $((T \times {}^\bullet s) \cup (s^\bullet \times T)) \cap F$ incident to s a tuple variable $\pi(s)$ is attached as a label;

tuple variables refer to normalized probability tuples $[x_1, \ldots, x_{n(s)}]$ *where*

$$0 \leq x_i \leq 1, \qquad \sum_{i=1}^{n(s)} x_i = 1;$$

$n(s)$ is a kind of "dimension" of s.

To all transitions $t \in T$ with ${}^\bullet t = \{a_1, \ldots, a_m\}, t^\bullet = \{b\}$, a conditional probability table $p(b|a_1, \ldots, a_m)$ is attached;

t is enabled *iff the input places ${}^\bullet t = \{a_1, \ldots, a_m\}$ are marked with probability tuples $\tau_1, \ldots, \tau_m$ and the output place $t^\bullet = \{b\}$ is empty;*

after firing *of t the input places $a_1, \ldots, a_m$ are cleared and the output place b is marked with $\pi(b) := (\pi(a_1) \times \ldots \times \pi(a_m)) \cdot p(b|a_1 \ldots a_m)$ where $\pi(a_1) := \tau_1, \ldots, \pi(a_m) := \tau_m$.*

If t is an input boundary transition, t is enabled *iff b is empty; $p(b)$ is a (prior) probability tuple and as such a $(1 \times n(b))$-matrix attached to t; after* firing t *$\pi(b) := p(b)$ has been put on b.*

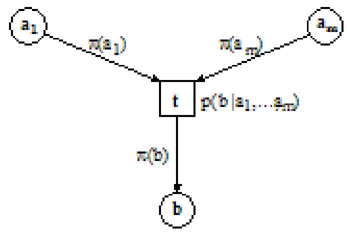

Figure 3. Matrix attached to transition t

If t is an output boundary transition, the respective tuple on a enabling *t is simply taken away when* firing t.

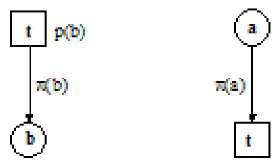

Figure 4. Boundary transitions

The high level net $U(N)$ defined that way is a high level dependency PN *based on N*

□

Even though probability tuples are flowing through U(N), it is not yet a probability propagation net because still the flows of evidences are not defined.

Remark 3.

Let r be a random variable; in the following we will use r_1 and r_0 as synonyms for r and $\neg r$.

□

Example 3.

The net U(DN) in figure 5 is the high level "upgrade" of the dependency Petri net DN in figure 2.

The matrices (probability tables) attached to the transitions are

$$p(a) = [a_1, a_0] = [0.01, 0.99], \qquad p(b) = [b_1, b_0] = [0.001, 0.999]$$

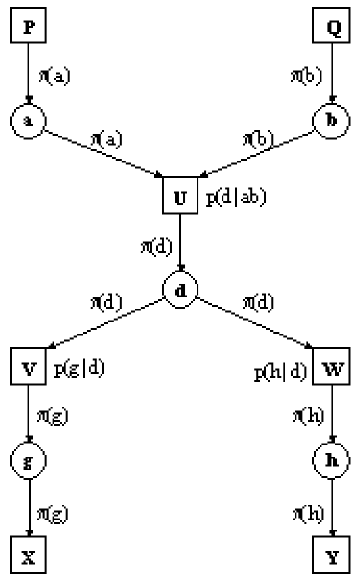

Figure 5. High level dependency Petri net U(DN)

$$
f_d(a,b) := p(d|ab) = \begin{array}{cc|cc}
\multicolumn{2}{c|}{f_d(a,b)} & \multicolumn{2}{c}{d} \\
\hline
a & b & 1 & 0 \\
\hline
1 & 1 & 0.99 & 0.01 \\
1 & 0 & 0.9 & 0.1 \\
0 & 1 & 0.5 & 0.5 \\
0 & 0 & 0.01 & 0.99
\end{array}
$$

$$
f_g(d) := p(g|d) = \begin{array}{c|cc}
f_g(d) & g_1 & g_0 \\
\hline
d_1 & 1.0 & 0.0 \\
d_0 & 0.2 & 0.8
\end{array}
$$

$$
f_h(d) := p(h|d) = \begin{array}{c|cc}
f_h(d) & h_1 & h_0 \\
\hline
d_1 & 0.2 & 0.8 \\
d_0 & 1.0 & 0.0
\end{array}
$$

The net U(DN) in figure 5 works as follows: The initial marking is empty. When transitions P and Q fire they put the probability tuples ((1×2)-matrices)

$P(a) = [0.01, 0.99]$ and $P(b) = [0.001, 0.999]$ on places a and b, respectively. Then transition U is enabled and can fire. Thereby places a and b are emptied and the probability tuple

$$\begin{aligned}\pi(d) &= (\pi(a) \times \pi(b)) \cdot p(d|ab)\\ &= [0.01 \cdot 0.001, 0.01 \cdot 0.999, 0.99 \cdot 0.001, 0.99 \cdot 0.999] \cdot p(d|ab)\\ &= [0.019, 0.981]\end{aligned}$$

is placed on d.

After firing of V d is empty and $\pi(g) = \pi(d) \cdot p(g|d) = [0.215, 0.785]$ is placed on g. By analogy, after firing of W d is empty and $\pi(h) = \pi(d) \cdot p(h|d) = [0.985, 0.015]$ is placed on h.

□

Now, some notations based on paths in dependency PNs are recommended.

Definition 6. [Path concepts]

Let $N = (P, T, F)$ be a dependency PN and $d \in P$, $U \in T$.

$Pa(d) := \{[(k_0, k_1), \ldots]|k_0 = d\}$ *is* the set of undirected paths starting at d∈ P*;*

$Pa(d, U) := \{[(k_0, k_1), \ldots]|k_0 = d, k_1 = U\}$ *is* the set of undirected paths starting with arc (d,U)*;*

$Pa(d, \neg U) := Pa(d) \setminus Pa(d, U)$ *is* the set of undirected paths starting at d∈ P but not with arc (d,U)*;*

$No(Pa(d, U))$ *is* the set of nodes in Pa(d,U)*;*

$No(Pa(d, \neg U))$ *is* the set of nodes in Pa(d,¬U)*;*

$U\uparrow := (No(Pa(d, U)) \cap P) \setminus \{d\}$ *is* set of places in Pa(d,U) without d*;*

$d\downarrow := (No(Pa(d, \neg U)) \cap P) \setminus \{d\}$ *is* set of places in Pa(d,¬U) without d.

□

Example 4. [See examples 2 and 3]

Let $N' = (P', T', F')$ be the underlying p/t-net in figures 2 and 5. Then we have

$$\begin{aligned}Pa(d) = \{&[(d, V)], [(d, V), (V, g)], [(d, V), (V, g), (g, X)],\\ &[(d, W)], [(d, W), (W, h)], [(d, W), (W, h), (h, Y)],\\ &[(d, U)], [(d, U), (U, a)], [(d, U), (U, a), (a, P)],\\ &[(d, U), (U, b)], [(d, U), (U, b), (b, Q)]\}\end{aligned}$$

$$\begin{aligned}Pa(d, U) = \{&[(d, U)], [(d, U), (U, a)], [(d, U), (U, a), (a, P)],\\ &[(d, U), (U, b)], [(d, U), (U, b), (b, Q)]\}\end{aligned}$$

$$\begin{aligned}Pa(d, \neg U) &= Pa(d) \setminus Pa(d, U)\\ &= \{[(d, V)], [(d, V), (V, g)], [(d, V), (V, g), (g, X)],\\ &\quad [(d, W)], [(d, W), (W, h)], [(d, W), (W, h), (h, Y)]\}\end{aligned}$$

$$\begin{aligned} No(Pa(d,U)) &= \{d,U,a,P,b,Q\} \\ No(Pa(d,\neg U)) &= \{d,V,g,X,W,h,Y\} \end{aligned}$$

$$\begin{aligned} U\!\uparrow := &\ (No(Pa(d,U)) \cap P^{'}) \setminus \{d\} \\ &= (\{d,U,a,P,b,Q\} \cap P^{'}) \setminus \{d\} \\ &= \{d,a,b\} \setminus \{d\} \\ &= \{a,b\} \end{aligned}$$

$$\begin{aligned} d\!\downarrow := &\ (No(Pa(d,\neg U)) \cap P^{'}) \setminus \{d\} \\ &= (\{d,V,g,X,W,h,Y\} \cap P^{'}) \setminus \{d\} \\ &= \{d,g,h\} \setminus \{d\} \\ &= \{g,h\} \end{aligned}$$

□

4 Transposed Probability Tables

The high level dependency PNs are far from being adequate to model besides the propagation of probabilities also the propagation of evidences (in form of likelihoods) and their mutual influencing. For that the dependency PNs have to be widened out considerably. Loosely speaking, for any transition t of the dependency PN as many transitions are added as there are transposes of the probability table attached to t.

So in this section we have to show a way how to calculate these transposes. We will do this using the running example of the preceding section.

Example 5. [Transposes of probability tables]

To transpose

$f_d(a,b) := p(d|ab) =$

$f_d(a,b)$		d	
a	b	1	0
1	1	0.99	0.01
1	0	0.9	0.1
0	1	0.5	0.5
0	0	0.01	0.99

we change this representation into a vector form $f_d(a,b)$:

d	a	b	$f_d(a,b)$	d	a	b	$f_a(b,d)$	d	a	b	$f_b(a,d)$
1	1	1	0.99	1	1	1	0.99	1	1	1	0.99
1	1	0	0.9	0	1	1	0.01	0	1	1	0.01
1	0	1	0.5	1	1	0	0.9	1	0	1	0.5
1	0	0	0.01	0	1	0	0.1	0	0	1	0.5
0	1	1	0.01	1	0	1	0.5	1	1	0	0.9
0	1	0	0.1	0	0	1	0.5	0	1	0	0.1
0	0	1	0.5	1	0	0	0.01	1	0	0	0.01
0	0	0	0.99	0	0	0	0.99	0	0	0	0.99

The vectors $f_d(a,b), f_a(b,d), f_b(a,d)$ differ in the role of the random variables a, b, d. In $f_d(a,b)$, d is dependent on a and b. This is visible by the fact that, viewed from the first row, 1's and 0's in column d change only once, in contrast to columns a and b. In $f_a(b,d)$ and $f_b(a,d)$, a and b, respectively, are the random variables with only one change in the 1's and 0's. $f_a(b,d)$ and $f_b(a,d)$ are built from $f_d(a,b)$ in three steps.

First, the dependent variables are chosen by inserting 1's and 0's with only one change into their respective columns. Second, the tuples (d,a,b) are completed such that the first independent variable (b and a, respectively) has three changes, and the second independent variable (d in both cases) has seven changes. Third, the probabilities are inserted in such a way that equal tuples (d,a,b) have equal probabilities (e. g.: 0 1 0 | 0.1).

The transposes are then obtained by transforming the vectors (back) into a matrix:

$f_a(b,d)$		a	
b	d	1	0
1	1	0.99	0.5
1	0	0.01	0.5
0	1	0.9	0.01
0	0	0.1	0.99

$f_b(a,d)$		b	
a	d	1	0
1	1	0.99	0.9
1	0	0.01	0.1
0	1	0.5	0.01
0	0	0.5	0.99

The order of the independent variables is arbitrary. But it has to equal the order of factors in the cross products:

$$\pi(d) = (\pi(a) \times \pi(b)) \cdot p(d|ab) = (\pi(a) \times \pi(b)) \cdot f_d(a,b)$$
$$\lambda(a) = (\pi(b) \times \lambda(d)) \cdot f_a(b,d)$$

$$\lambda(b) = (\pi(a) \times \lambda(d)) \cdot f_b(a, d) \text{ (see next chapter)}$$

$f_a(b, d)$ and $f_b(a, d)$ are not the only transposes of $f_d(a, b)$. Also the matrices $f_a(d, b)$ and $f_b(d, a)$ can be constructed in the same way and require the right order of factors in the corresponding cross products.

If only two random variables are involved the transposed tables are obtained by the usual way of interchanging rows and columns.

$$f_d(g) := (f_g(d))^t = (p(g|d))^t$$
$$f_d(h) := (f_h(d))^t = (p(h|d))^t$$

□

Next, we aim at constructing the complete probability propagation nets.

5 Probability Propagation Nets

Now we are well equipped to construct probability propagation nets (PPNs). In fact, the task consists of two parts: to get the net structure and to obtain the inscriptions for places, transitions, and arcs. The net structure will be reused for the propagation of mass values - then with different inscriptions (see next chapter).

We will continue studying the dependency PNs of examples 2 and 3. Of course, this is a special case, but it is not necessary to consider the general form of a dependency PN. This is certainly more complicated, but by no means more complex. By help of the example, we will easily understand how PPNs are to be constructed on the basis of dependency PNs.

We will proceed as follows. The formula (see [KSH91], p.353, last lines)

$$\pi(d|E) = \frac{1}{c} \sum_{d\downarrow} \sum_{U\uparrow} \left[\lambda_E\left(d\downarrow\right) \cdot \pi\left(d\downarrow|d\right) \cdot \lambda_E\left(d\right) \cdot \pi\left(d|ab\right) \cdot \lambda_E\left(U\uparrow\right) \cdot \pi\left(U\uparrow\right)\right] \quad (1)$$

describes the probabilities we have to deal with. The main task will be to transform multiplications and summations of formula (1) into matrix operations (as part of the transition firings).

Example 6. [See examples 2, 3, 4]

We will confine ourselves to the standards of Bayesian networks, i.e. influences will only flow "upwards from the bottom". So, we omit the user defined influences (observations) $\lambda_E(d)$ and $\lambda_E(U\uparrow)$. In addition, we will omit all subscripts $_E$. $\frac{1}{c}$ is for normalizing the probability tuples. It will also be omitted for simplification.

The formula (1) refers to nodes U and d. This is the 1st case for which (1) will be transformed into that part of the PPN which is to calculate $\pi(d|E)$ and to demonstrate the net dynamics of the calculation.

By means of the nodes P and a, as 2nd case, we gain the subnet for $\pi(a|E)$ etc. Thus, the PPN grows until it describes all flows of probabilities and evidences (likelihoods) completely. The final PPN is not only a transparent picture of these flows and their mutual influences, it also elucidates the formula with the help of the Petri net dynamics.

1st case:

The formula

$$\pi(d|E) = \sum_{d\downarrow} \sum_{U\uparrow} \left[\lambda\left(d\downarrow\right) \cdot \pi\left(d\downarrow|d\right) \cdot \pi\left(d|ab\right) \cdot \pi\left(U\uparrow\right)\right] \tag{2}$$

will be transformed for $d\downarrow = \{g, h\}$ and $U\uparrow = \{a, b\}$. Notice that the probabilities of a and b as well as the evidences of g and h are considered independent. So we have

$$\begin{aligned}
\pi(d|E) &= \sum_{g,h} \sum_{a,b} \left[\lambda\left(d\downarrow\right) \cdot \pi\left(d\downarrow|d\right) \cdot \pi\left(d|ab\right) \cdot \pi\left(U\uparrow\right)\right] \\
&= \sum_{g,h} \sum_{a,b} \left[\lambda\left(gh\right) \cdot \pi\left(gh|d\right) \cdot \pi\left(d|ab\right) \cdot \pi\left(ab\right)\right] \\
&= \sum_{g,h} \sum_{a,b} \left[\lambda\left(g\right) \cdot \lambda\left(h\right) \cdot \pi\left(g|d\right) \cdot \pi\left(h|d\right) \cdot \pi\left(d|ab\right) \cdot \pi\left(a\right) \cdot \pi\left(b\right)\right] \\
&= \left[\sum_{g} \lambda\left(g\right) \cdot \pi\left(g|d\right)\right] \cdot \left[\sum_{h} \lambda\left(h\right) \cdot \pi\left(h|d\right)\right] \cdot \left[\sum_{a,b} \pi\left(d|ab\right) \cdot \pi\left(a\right) \cdot \pi\left(b\right)\right]
\end{aligned} \tag{3}$$

Now, we will switch to a vector and matrix notation, where we interpret the sums as belonging to matrix products:

$\cdot$ denotes the original products of the formula,
$\circ$ denotes a componentwise product
$(a_1, a_2) \circ (b_1, b_2) = (a_1 b_1, a_2 b_2)$,
$\times$ denotes a cartesian (cross) product
$(a_1, a_2) \times (b_1, b_2) = (a_1 b_1, a_1 b_2, a_2 b_1, a_2 b_2)$,
$*$ denotes the usual matrix product

This leads to the following form of the formula (3)

$$\pi(d|E) = \left[\underbrace{(\lambda(g) * f_d(g))}_{=:\lambda_g(d)} \circ \underbrace{(\lambda(h) * f_d(h))}_{=:\lambda_h(d)} \right] \circ \left[\underbrace{(\pi(a) \times \pi(b)) * f_d(a,b)}_{=:\pi(d)} \right]$$

$$= [\lambda_g(d) \circ \lambda_h(d)] \circ \pi(d) \quad = \quad \underbrace{\lambda(d) \circ \pi(d)}_{=:bel(d)}$$

Thus, as *1st case*, we showed that $\pi(d|E) = bel(d)$ holds ("belief" of d is a notion in the field of Bayesian networks; not to mix up with the notion of belief in the next chapter). We also constructed the net in figure 6 as that part of the entire PPN which belongs to node d in figure 5.

In the upper part of the net in figure 6, the (1×2)-matrices $\pi(a) = [\pi(a_1), \pi(a_0)]$ and $\pi(b) = [\pi(b_1), \pi(b_0)]$ are put on places a and b, respectively, by firing of the corresponding boundary transitions $\pi(a)$ and $\pi(b)$. Thereafter, the row vector $\pi(a) \times \pi(b)$ is multiplied by matrix $f_d(a,b)$ when the corresponding transition $f_d(a,b)$ fires; in addition places a and b are emptied and the result $\pi(d)$ is put on place d. (Because of clearness, some indices are saved and so some names of places are repeated.)

In the lower part of the net in figure 6, the (1×2)-matrices $\lambda(g) = [\lambda(g_1), \lambda(g_0)]$ and $\lambda(h) = [\lambda(h_1), \lambda(h_0)]$ are put on places g and h, respectively, by firing of the corresponding transitions $\lambda(g)$ and $\lambda(h)$. By firing of the transitions $f_d(g)$ and $f_d(h)$ places g and h are emptied and the row vectors $\lambda_g(d) = \lambda(g) * f_d(g)$ and $\lambda_h(d) = \lambda(h) * f_d(h)$ are put on "their" respective places d. Then the transition $m_d(g,h)$ empties the places d and puts the row vector $\lambda(d) = \lambda_g(d) \circ \lambda_h(d)$ on d. So, $(vector * matrix)$-multiplications and $(vector \circ vector)$-multiplications are carried out by "*f*- and *m-transitions*", respectively.

Throughout all net figures, we do without representing the beliefs $bel(\ldots) = \pi(\ldots) \circ \lambda(\ldots)$ in favor of keeping the figures as clear as possible. The transitions without inscriptions are simply to complete the reproductions of the empty marking.

2nd case:

Next, we transform formula (2) for nodes P and a.

$$\pi(a|E) = \sum_{a\downarrow} \sum_{P\uparrow} [\lambda\,(d\downarrow) \cdot \pi\,(d\downarrow|d) \cdot \pi\,(d|ab) \cdot \pi\,(U\uparrow)]$$

where $a\downarrow = \{b, d, g, h\}$, $P\uparrow = \emptyset$, $d\downarrow = \{g, h\}$, $U\uparrow = \{a, b\}$.

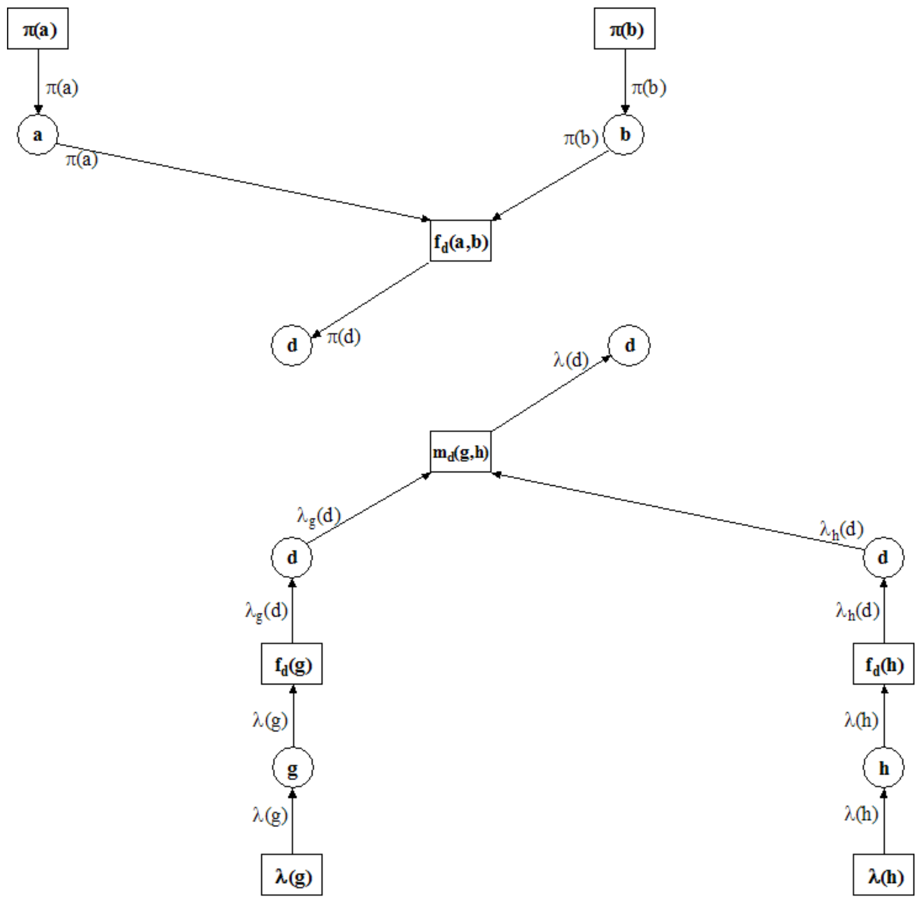

Figure 6. Sub-PPN of the 1st case

$$\pi(a|E) = \sum_{b,d,g,h} \left[\lambda\left(d{\downarrow}\right) \cdot \pi\left(d{\downarrow}|d\right) \cdot \pi\left(d|ab\right) \cdot \pi\left(U{\uparrow}\right)\right]$$

$$= \sum_{b,d,g,h} \left[\lambda\left(gh\right) \cdot \pi\left(gh|d\right) \cdot \pi\left(d|ab\right) \cdot \pi\left(ab\right)\right]$$

$$= \sum_{b,d,g,h} \left[\lambda\left(g\right) \cdot \lambda\left(h\right) \cdot \pi\left(g|d\right) \cdot \pi\left(h|d\right) \cdot \pi\left(d|ab\right) \cdot \pi\left(a\right) \cdot \pi\left(b\right)\right]$$

$$= \pi\left(a\right) \cdot \sum_{b,d} \left[\left[\sum_{g} \lambda\left(g\right) \cdot \pi\left(g|d\right)\right] \cdot \left[\sum_{h} \lambda\left(h\right) \cdot \pi\left(h|d\right)\right] \cdot \left[\pi\left(b\right) \cdot \pi\left(d|ab\right)\right]\right]$$

$$= \pi\left(a\right) \cdot \sum_{b,d} \left[\left[\underbrace{\left[\lambda\left(g\right) * f_d(g)\right]}_{=:\lambda_g(d)} \circ \underbrace{\left[\lambda\left(h\right) * f_d(h)\right]}_{=:\lambda_h(d)}\right] \cdot \left[\pi\left(b\right) \cdot \pi\left(d|ab\right)\right]\right]$$

$$= \pi\left(a\right) \cdot \sum_{b,d} \left[\underbrace{\left[\lambda_g(d) \circ \lambda_h(d)\right]}_{=:\lambda(d)} \cdot \left[\pi\left(b\right) \cdot \pi\left(d|ab\right)\right]\right]$$

$$= \pi\left(a\right) \cdot \sum_{b,d} \lambda(d) \cdot \left[\pi\left(b\right) \cdot \pi\left(d|ab\right)\right]$$

$$= \pi\left(a\right) \circ \left[\left[\pi(b) \times \lambda(d)\right] \cdot \pi\left(d|ab\right)\right]$$

$$= \pi\left(a\right) \circ \left[\left[\pi(b) \times \lambda(d)\right] * f_a(b,d)\right]$$

$$= \pi\left(a\right) \circ \lambda(a) = bel(a)$$

The corresponding sub-PPN is shown in figure 7.

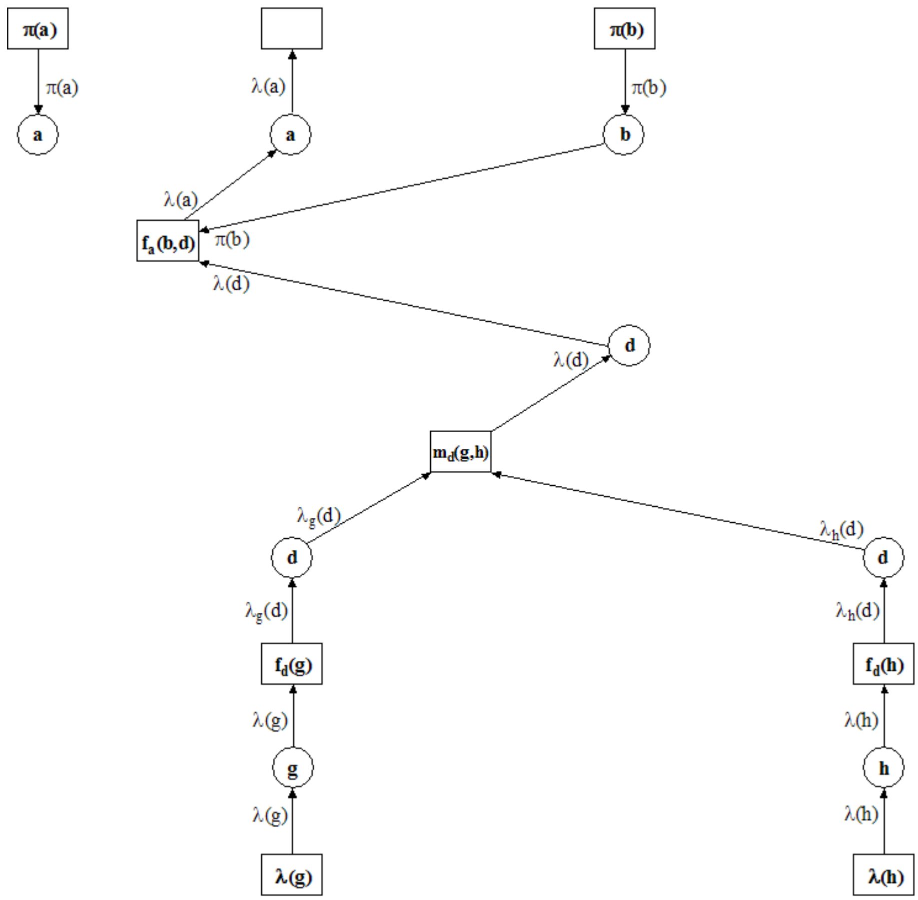

Figure 7. Sub-PPN of the 2nd case

3rd case

For Q and b the formula (2) is transformed into

$$\pi(b|E) = \sum_{b\downarrow} \sum_{Q\uparrow} \left[\lambda\left(d\downarrow\right) \cdot \pi\left(d\downarrow|d\right) \cdot \pi\left(d|ab\right) \cdot \pi\left(U\uparrow\right)\right]$$

where $b\downarrow = \{a, d, g, h\}$, $Q\uparrow = \emptyset$, $d\downarrow = \{g, h\}$, $U\uparrow = \{a, b\}$.

By symmetry to the 2nd case (P and a), we find

$$\pi(b|E) = \pi(b) \circ \lambda(b) = \ bel(b)$$

The corresponding sub-PPN is shown in figure 8.

4th case

For V and g the formula (2) is transformed into

$$\pi(g|E) = \sum_{g\downarrow} \sum_{V\uparrow} \left[\lambda\left(d\downarrow\right) \cdot \pi\left(d\downarrow|d\right) \cdot \pi\left(d|ab\right) \cdot \pi\left(U\uparrow\right)\right]$$

where $g\downarrow = \emptyset$, $V\uparrow = \{a, b, d, h\}$, $d\downarrow = \{g, h\}$, $U\uparrow = \{a, b\}$.

$$\pi(g|E) = \sum_{a,b,d,h} \left[\lambda\left(gh\right) \cdot \pi\left(gh|d\right) \cdot \pi\left(d|ab\right) \cdot \pi\left(ab\right)\right]$$

$$= \sum_{a,b,d,h} \left[\lambda\left(g\right) \cdot \lambda\left(h\right) \cdot \pi\left(g|d\right) \cdot \pi\left(h|d\right) \cdot \pi\left(d|ab\right) \cdot \pi\left(a\right) \cdot \pi\left(b\right)\right]$$

$$= \sum_{a,b,d,h} \left[\left[\lambda\left(g\right) \cdot \pi\left(g|d\right)\right] \cdot \left[\lambda\left(h\right) \cdot \pi\left(h|d\right)\right] \cdot \left[\pi\left(a\right) \cdot \pi\left(b\right) \cdot \pi\left(d|ab\right)\right]\right]$$

$$= \sum_{a,b,d} \left[\left[\lambda\left(g\right) \cdot \pi\left(g|d\right)\right] \cdot \left[\sum_{h} \lambda\left(h\right) \cdot \pi\left(h|d\right)\right] \cdot \left[\pi\left(a\right) \cdot \pi\left(b\right) \cdot \pi\left(d|ab\right)\right]\right]$$

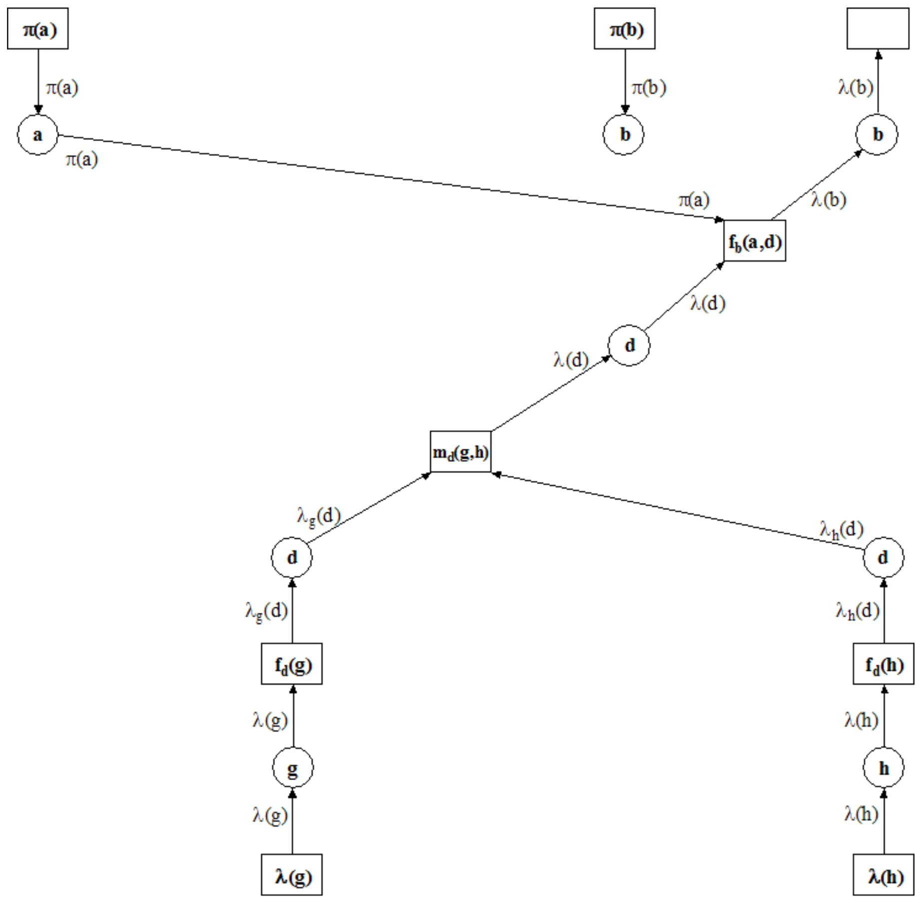

Figure 8. Sub-PPN of the 3rd case

$$= \sum_{a,b,d} \left[[\lambda(g) \cdot \pi(g|d)] \cdot \underbrace{[\lambda(h) * f_d(h)]}_{=:\lambda_h(d)} \cdot [\pi(a) \cdot \pi(b) \cdot \pi(d|ab)] \right]$$

$$= \sum_{d} \left[\left[\sum_{a,b} \pi(a) \cdot \pi(b) \cdot \pi(d|ab) \right] \cdot \lambda_h(d) \cdot [\lambda(g) \cdot \pi(g|d)] \right]$$

$$= \sum_{d} \left[\underbrace{[[\pi(a) \times \pi(b)] * f_d(a,b)]}_{=:\pi(d)} \cdot \lambda_h(d) \cdot [\lambda(g) \cdot \pi(g|d)] \right]$$

$$= \left[\sum_{d} \underbrace{[\pi(d) \circ \lambda_h(d)]}_{=:\pi_g(d)} \cdot \pi(g|d) \right] \cdot \lambda(g)$$

$$= \underbrace{[\pi_g(d) * f_g(d)]}_{=:\pi(g)} \circ \lambda(g) = \ bel(g)$$

The corresponding sub-PPN is shown in figure 9.

5th case

For W and h the formula (2) is transformed into

$$\pi(H|E) = \sum_{h\downarrow} \sum_{W\uparrow} [\lambda(d\downarrow) \cdot \pi(d\downarrow|d) \cdot \pi(d|ab) \cdot \pi(U\uparrow)]$$

where $h\downarrow = \emptyset$, $W\uparrow = \{a,b,d,g\}$, $d\downarrow = \{g,h\}$, $U\uparrow = \{a,b\}$.

By symmetry to the 4th case (V and g) we find

$$\pi(h|E) = \pi(h) \circ \lambda(h) = bel(h).$$

The corresponding subnet in figure 10 results mainly from replacing $m_g(h,d)$ and $f_g(d)$ by $m_h(g,d)$ and $f_h(d)$, respectively.

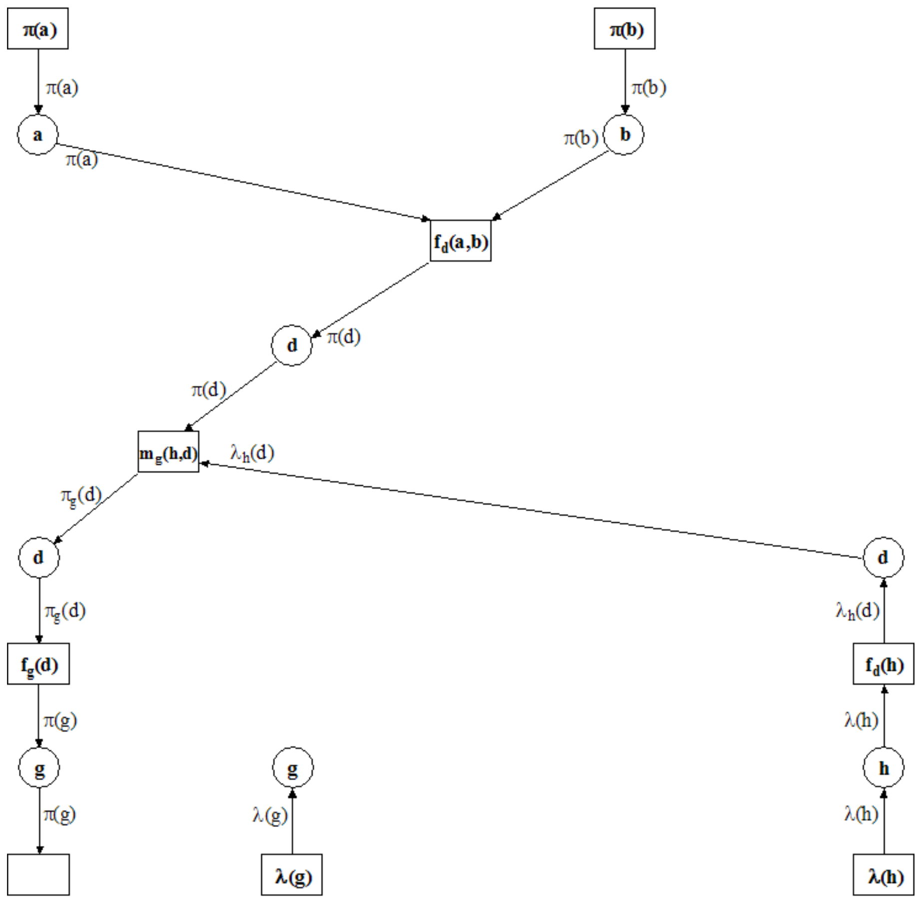

Figure 9. Sub-PPN of the 4th case

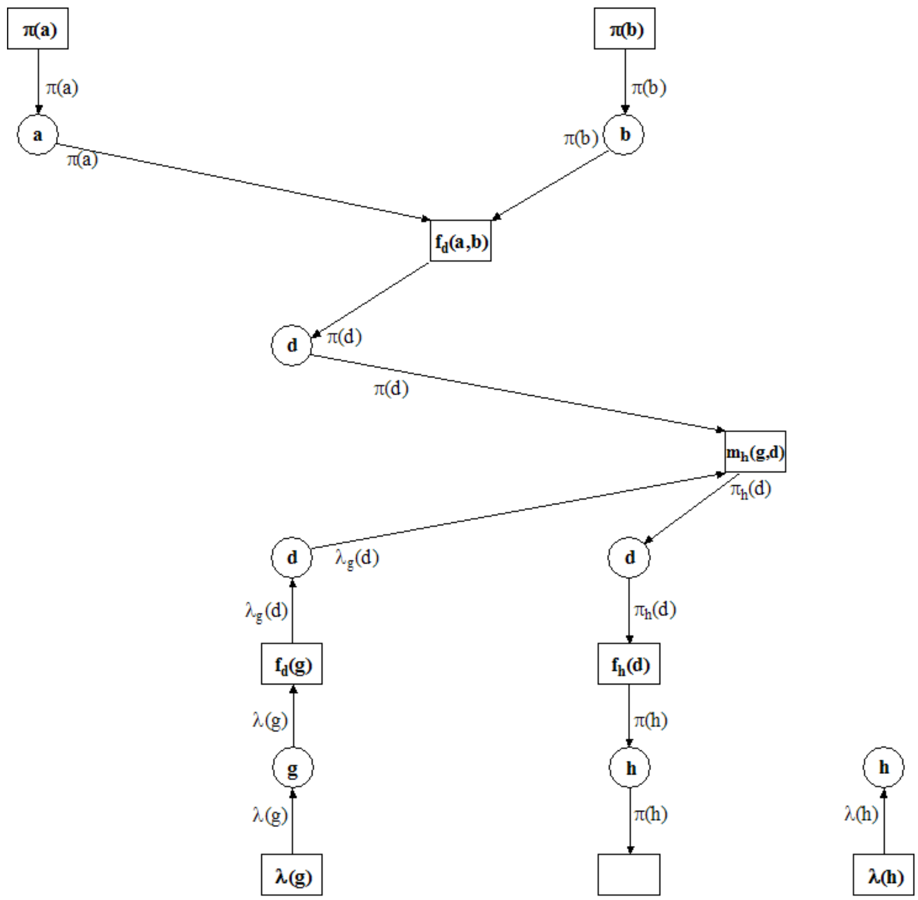

Figure 10. Sub-PPN of the 5th case

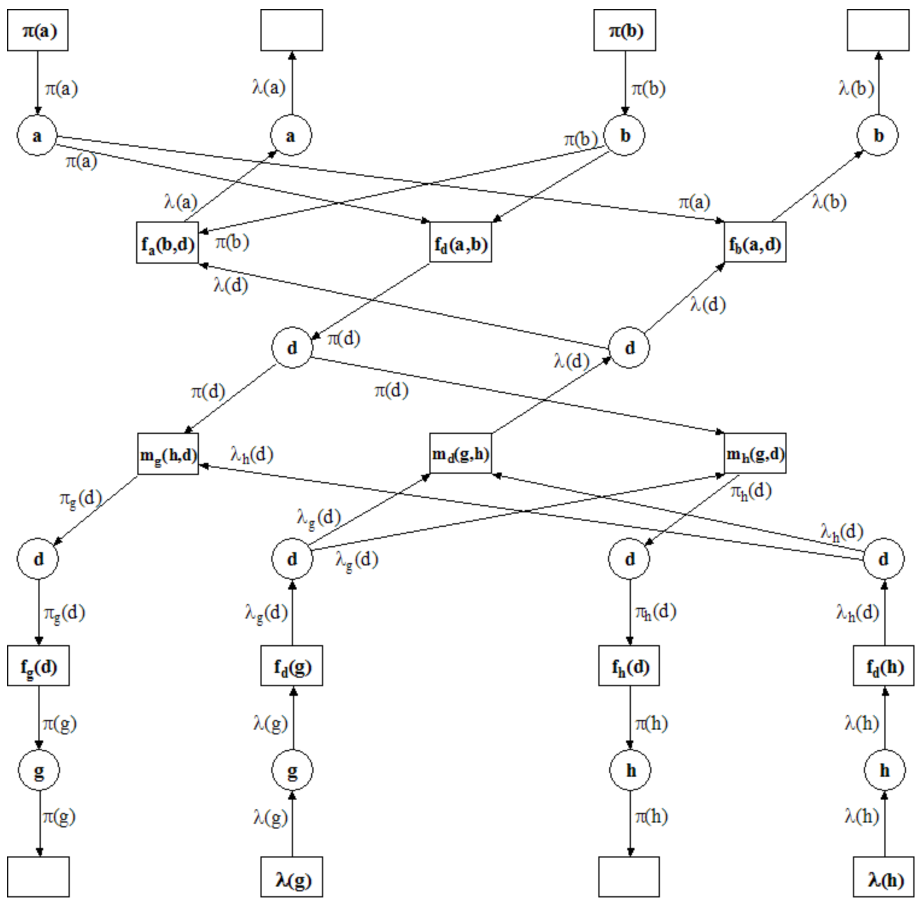

Figure 11. The entire PPN

The entire PPN as the union of the five sub-PPNs is shown in figure 11. In order to present some results, we will take the same input values as in example 3:

$$\pi(a) = [0.01, 0.99],\ \pi(b) = [0.001, 0.999].$$

In addition, we have to formulate the fact that there is absolutely no evidence for g and h being true or false. So we assume

$$\lambda(g) = \lambda(h) = [1, 1].$$

It is common practice to use [1,1] for evidences (instead of [0.5,0.5]) even though the tuple is not normalized.

The boundary transitions at the bottom put $\lambda(g) = \lambda(h) = [1, 1]$ on the respective places g and h. Then the transitions $f_d(g)$ and $f_d(h)$ are enabled and fire, thereby emptying their input places and putting

$$\lambda_g(d) = \lambda(g) * f_d(g) = [1, 1] \text{ and } \lambda_h(d) = \lambda(h) * f_d(h) = [1, 1]$$

on the respective places d.

After firing of the upper boundary transitions $\pi(a)$ and $\pi(b)$ and of $f_d(a, b)$, $\pi(d) = [0.019, 0, 981]$ is put on the respective place d. Now, the transition $m_g(h, d)$ is enabled by tuples $\lambda_h(d) = [1, 1]$ and $\pi(d)$. Firing of $m_g(h, d)$ results in $\pi_g(d) = \pi(d)$ which enables $f_g(d)$.

$$\pi_g(d) * f_g(d) = [0.215, 0.785] = \pi(g)$$

is the result on place g. Similarly, $\pi(h) = [0.985, 0.015]$ is the result on place h. Finally, the empty marking is completely reproduced by firing of the unnamed output transitions.

These values are the same ones we obtained in example 3. In summary, we make out that the dependency PN U(DN) has the same modeling power as the corresponding PPN without any additional evidence.

For the input values

$$\pi(a) = [0.01, 0.99], \pi(b) = [0.001, 0.999], \lambda(g) = [1.0, 0.0], \lambda(h) = [0.0, 1, 0]$$

we find

$$bel(d) = [1.0, 0.0], bel(a) = [0.4647, 0.5353], bel(b) = [0.026, 0.974].$$

□

Even though this example shows only a special case, it should be clear now how to get the PPNs for larger dependency PNs, for higher degrees of branching (on the input side of transitions and on the output side of places), for tuples "longer" than pairs, etc.

6 Mass Distributions

This chapter is to introduce into the field of mass distributions and their PN based propagations.

Mass distributions have a poorer structure than probability distributions. In contrast to probability functions, mass functions are not monotonous and therefore not additive. There is no value relation at all between sets and subsets. This gives more freedom for modeling, but makes calculations difficult and - as a consequence - the propagation of mass tuples. This lack of structure permits only one set system - the power set where sets and subsets are of equal rank.

Definition 7.

Let Ω be a non-empty set (usually called "frame of discernment"*);*

every function $m : 2^{\Omega} \to [0,1]$ is a mass distribution *iff*

$$m(\emptyset) = 0 \quad \textit{and} \quad \sum_{A \subseteq \Omega} m(A) = 1 \quad \textit{hold};$$

$A \subseteq \Omega$ *is a* focal element *of m iff $m(A) > 0$.* □

$m(A)$ can be understood as the degree of support, reliance, confidence, etc. which we (or some expert) grant A without being forced to grant *A's complement* the value $1-m(A)$, or to distribute $m(A)$ among *A's* elements (as in probability theory). m is sort of a generalized probability (in [Sh76] Shafer calls it a *basic probability assignment*). In contrast to probability functions, mass functions allow to specify our degree of ignorance.

Definition 8.

Let Ω be a frame of discernment, $A \subseteq \Omega$, and let m_I and m_B be two mass distributions;

complete ignorance *is expressed by the mass distribution*

$$m_I(A) = \begin{cases} 0 & \textit{if} \quad A \subset \Omega \\ 1 & \textit{if} \quad A = \Omega \end{cases}$$

a Bayesian mass distribution *m_B is defined by*

$$m_B(A) > 0 \quad \textit{iff} \quad |A| = 1$$

for $\Omega = \{\omega_1, \ldots, \omega_n\}$, *$P(\omega_i) := m_B(\omega_i),\ 1 \leq i \leq n$, defines a* discrete probability distribution*.*

□

Definition 9.

Let m be a mass distribution on Ω;
the function

$$Bel_m : 2^{\Omega} \to [0,1] \qquad Bel_m(A) = \sum_{B \subseteq A} m(B)$$

is the belief function *based on m;*

the function

$$Pl_m : 2^{\Omega} \to [0,1] \qquad Pl_m(A) = \sum_{B \cap A \neq \emptyset} m(B)$$

is the plausibility function *based on m;*

the function

$$Dbt_m : 2^{\Omega} \to [0,1] \qquad Dbt_m(A) = \sum_{B \cap A = \emptyset} m(B)$$

is the doubt function *based on m;*

the function

$$Unc_m : 2^{\Omega} \to [0,1] \qquad Unc_m(A) = \sum_{B \cap A \neq \emptyset \wedge B \not\subseteq A} m(B)$$

is the uncertainty function *based on m.*

□

$Bel_m(A)$ measures the evidence supporting A (the credibility of A: everything that speaks in A's favor).

$Pl_m(A)$ measures the evidence supporting A under the most favorable circumstances (everything that does not speak against A).

Proposition 1.

Let be $A \subseteq \Omega$ *and m a mass distribution; then the following holds:*

$$\begin{array}{rlll}
Pl_m(A) & = 1 - Bel_m(\overline{A}) & & (duality) \\
Bel_m(\emptyset) & = Pl_m(\emptyset) & = 0 & \\
Bel_m(\Omega) & = Pl_m(\Omega) & = 1 & \\
Bel_m(A) & \leq Pl_m(A) & & \text{for all } A \subseteq \Omega \\
Bel_m(A) & + Bel_m(\overline{A}) & \leq 1 & \text{for all } A \subseteq \Omega \\
Pl_m(A) & + Pl_m(\overline{A}) & \geq 1 & \text{for all } A \subseteq \Omega \\
Bel_m(A) + Dbt_m(A) & + Unc_m(A) & = 1 & \text{for all } A \subseteq \Omega
\end{array}$$

For m_I and every Bayesian mass distribution m_B the following holds:

$$\begin{array}{lll} Bel_{m_I}(A) + Bel_{m_I}(\overline{A}) & = 0 & \text{for } \emptyset \subset A \subset \Omega \\ Pl_{m_I}(A) + Pl_{m_I}(\overline{A}) & = 1 & \text{for } \emptyset \subset A \subset \Omega \end{array}$$

$$\begin{array}{rll} & Bel_{m_B} = Pl_{m_B} & \text{(self-duality)} \\ P := & Bel_{m_B} = Pl_{m_B} & \text{is a discrete probability function} \end{array}$$

□

Our next point is the propagation of masses. We want to show that the net structure of PPNs with a new kind of matrices is a workable concept for the modeling of mass flows. Following R. Kruse et al. (see [KNK91,KS90,KSH91,KSK91]), we consider specialization matrices, a generalization of conditional probability tables.

Definition 10. *Let Ω be a frame of discernment;*

(1) a matrix $V : 2^\Omega \times 2^\Omega \to [0,1]$ is a specialization matrix *iff*

$$\sum_{B \subseteq \Omega} V[A,B] = 1 \;\; for \;\; all \;\; A \subseteq \Omega \quad and$$

$$B \not\subseteq A \Rightarrow V[A,B] = 0$$

(2) let V be a specialization matrix and m a mass distribution on 2^Ω;

$$m \odot V \;\; is \;\; defined \;\; by$$

$$(m \odot V)[B] := \begin{cases} \frac{1}{c} \cdot \sum\limits_{A \subseteq \Omega} m[A] \cdot V[A,B] & if \;\; B \neq \emptyset \\ 0 & otherwise \end{cases}$$

$$for \;\; all \;\; B \subseteq \Omega \;\; and \;\; c := \sum_{A \subseteq \Omega} \sum_{B \neq \emptyset} m[A] \cdot V[A,B] > 0$$

the mass distribution $m' = m \odot V$ is a specialization *of m.*

□

Example 7. [see example 3]

In net $U(DN)$ (figure 5, example 3), the matrix (conditional probability table) $p(g|d)$ attached to transition V is

$$f_g(d) := p(g|d) = \begin{array}{c|cc} f_g(d) & g & \neg g \\ \hline d & 1.0 & 0.0 \\ \neg d & 0.2 & 0.8 \end{array}$$

and the corresponding probability distribution is $\pi(d) = [0.019, 0.981]$.

For $\Omega = \Omega^d \times \Omega^g = \{d, \neg d\} \times \{g, \neg g\} = \{(d, g), (d, \neg g), (\neg d, g), (\neg d, \neg g)\}$ the following matrix consists of a specialization matrix V and a mass distribution $m[d]$ as last column (suitable to $p(g|d)$ and $\pi(d)$); the product $m' = m \odot V$ is shown in the last row.

$V =$

A \ B	$\emptyset$	$\{d\} \times \{g\}$	$\{d\} \times \{\neg g\}$	$\{d\} \times \Omega^g$	$\{\neg d\} \times \{g\}$	$\{\neg d\} \times \{\neg g\}$	$\{\neg d\} \times \Omega^g$	$\Omega^d \times \{g\}$	$\Omega^d \times \{\neg g\}$	$\Omega^d \times \Omega^g$	$m[d]$ as column
$\emptyset$	1										0
$\{d\} \times \{g\}$		1									0
$\{d\} \times \{\neg g\}$			1								0
$\{d\} \times \Omega^g$		1.0	0.0	0							0.019
$\{\neg d\} \times \{g\}$					1						0
$\{\neg d\} \times \{\neg g\}$						1					0
$\{\neg d\} \times \Omega^g$					0.2	0.8	0				0.981
$\Omega^d \times \{g\}$								1			0
$\Omega^d \times \{\neg g\}$									1		0
$\Omega^d \times \Omega^g$										1	0
$m \odot V$	0	0.019	0	0	0.1962	0.7848	0	0	0	0	

The elements not shown in V equal zero.

(1) V is a specialization matrix since all row sums are equal to 1 and $V[A, B] \neq 0 \Rightarrow B \subseteq A$ holds;

this is true for the non-zero elements on the main diagonal and for
$V[\{d\} \times \Omega^g, \{d\} \times \{g\}] = 1$,
$V[\{\neg d\} \times \Omega^g, \{\neg d\} \times \{g\}] = 0.2$,
$V[\{\neg d\} \times \Omega^g, \{\neg d\} \times \{\neg g\}] = 0.8$.

(2) $c = \sum_A m[A] \cdot \sum_{B \neq \emptyset} V[A, B] = 0.019 \cdot 1.0 + 0.981 \cdot (0.2 + 0.8) = 1$

$$(m \odot V)[B] = \sum_A m[A] \cdot V[A, B] \quad \text{if} \quad B \neq \emptyset \quad \text{so}$$

$$\begin{array}{lclcl}
(m \odot V)[\{d\} \times \{g\}] & = & m[\{d\} \times \Omega^g] \cdot 1 & = & 0.019 \\
(m \odot V)[\{\neg d\} \times \{g\}] & = & m[\{\neg d\} \times \Omega^g] \cdot 0.2 & = & 0.1962 \\
(m \odot V)[\{\neg d\} \times \{\neg g\}] & = & m[\{\neg d\} \times \Omega^g] \cdot 0.8 & = & 0.7848
\end{array}$$

The elements of V indicate the most important aspect of the specialization, namely that the relative portions of masses flow from sets to their subsets. For example, 0.2 and 0.8 are the relative shares of

$$B_1 = \{\neg d\} \times \{g\} \text{ and } B_2 = \{\neg d\} \times \{\neg g\} \text{ in the mass}$$

$$m[A] = m[\{d\} \times \Omega_g] = 0.981 \text{ of A} \quad (B_1, B_2 \subseteq A).$$

The specialization matrix can be represented by a so-called "orthogonal extension matrix" V':

$$\begin{array}{l}
V' : 2^{\Omega^d} \times 2^{\Omega^g} \to [0, 1] \\
V'[A', B'] := V[A' \times \Omega^g, A' \times B'].
\end{array}$$

This yields for example:

$$\begin{array}{ll}
V'[\{d\}, \{g\}] & = V[\{d\} \times \Omega^g, \{d\} \times \{g\}] \qquad = 1.0 \\
V'[\{\neg d\}, \{\neg g\}] & = V[\{\neg d\} \times \Omega^g, \{\neg d\} \times \{\neg g\}] = 0.8;
\end{array}$$

The entire matrix V' is

V	$\emptyset^g$	$\{g\}$	$\{\neg g\}$	Ω^g
$\emptyset^d$	0	0	0	0
$\{d\}$	0	1	0	0
$\{\neg d\}$	0	0.2	0.8	0
Ω^d	0	0	0	1

The suitable mass distributing corresponding to m is

$$m' : 2^{\Omega^d} \to [0, 1] \qquad \begin{array}{l} m'[\emptyset^d] := 0 \\ m'[A'] := m[A' \times \Omega^g]; \end{array}$$

altogether we have:

$$\begin{array}{lcl} m'[\emptyset^d] & = & 0.0 \\ m'[\{d\}] & = & 0.019 \\ m'[\{\neg d\}] & = & 0.981 \\ m'[\Omega^d] & = & 0.0 \end{array}$$

which yields

$$\begin{array}{lcl} (m' \odot V')[\emptyset^g] & = & 0.0 \\ (m' \odot V')[\{g\}] & = & 0.215 \\ (m' \odot V')[\{\neg g\}] & = & 0.785 \\ (m' \odot V')[\Omega^g] & = & 0.0 \end{array}$$

m' and its specialization $m' \odot V'$ are the Bayesian mass distributions belonging to the probabilities $\pi(d) = [0.019, 0.981]$ and $\pi(g) = \pi(d) * f_g(d) = [0.215, 0.981]$.

This gives rise to formulate the result:

The PPN in figure 11 is able to propagate masses if we re-define all inscriptions as follows:

$$\begin{array}{lcl lcl} \pi(a)[\emptyset^a] & = & 0.0 & \pi(b)[\emptyset^b] & = & 0.0 \\ \pi(a)[\{a\}] & = & 0.01 & \pi(b)[\{b\}] & = & 0.001 \\ \pi(a)[\{\neg a\}] & = & 0.99 & \pi(b)[\{\neg b\}] & = & 0.999 \\ \pi(a)[\Omega^a] & = & 0.0 & \pi(b)[\Omega^b] & = & 0.0 \\ & & & & & \\ \lambda(g)[\emptyset^g] & = & 0.0 & \lambda(h)[\emptyset^h] & = & 0.0 \\ \lambda(g)[\{g\}] & = & 0.5 & \lambda(h)[\{h\}] & = & 0.5 \\ \lambda(g)[\{\neg g\}] & = & 0.5 & \lambda(h)[\{\neg h\}] & = & 0.5 \\ \lambda(g)[\Omega^g] & = & 0.0 & \lambda(h)[\Omega^h] & = & 0.0 \end{array}$$

$f_g(d)$	$\emptyset^g$	$\{g\}$	$\{\neg g\}$	Ω^g
$\emptyset^d$	0	0	0	0
$\{d\}$	0	1.0	0.0	0
$\{\neg d\}$	0	0.2	0.8	0
Ω^d	0	0	0	1

$f_h(d)$	$\emptyset^h$	$\{h\}$	$\{\neg h\}$	Ω^h
$\emptyset^d$	0	0	0	0
$\{d\}$	0	0.2	0.8	0
$\{\neg d\}$	0	1.0	0.0	0
Ω^d	0	0	0	1

$f_d(a,b)$	$\emptyset^d$	$\{d\}$	$\{\neg d\}$	Ω^d	$\pi(a) \times \pi(b)$ as column
$\emptyset^a \times \emptyset^b$	0	0	0	0	0
$\emptyset^a \times \{b\}$	0	0	0	0	0
$\emptyset^a \times \{\neg b\}$	0	0	0	0	0
$\emptyset^a \times \Omega^b$	0	0	0	0	0
$\{a\} \times \emptyset^b$	0	0	0	0	0
$\{a\} \times \{b\}$	0	0.99	0.01	0	0.00001
$\{a\} \times \{\neg b\}$	0	0.9	0.1	0	0.99999
$\{a\} \times \Omega^b$	0	0	0	0	0
$\{\neg a\} \times \emptyset^b$	0	0	0	0	0
$\{\neg a\} \times \{b\}$	0	0.5	0.5	0	0.00099
$\{\neg a\} \times \{\neg b\}$	0	0.01	0.99	0	0.00999
$\{\neg a\} \times \Omega^b$	0	0	0	0	0
$\Omega^a \times \emptyset^b$	0	0	0	0	0
$\Omega^a \times \{b\}$	0	0	0	0	0
$\Omega^a \times \{\neg b\}$	0	0	0	0	0
$\Omega^a \times \Omega^b$	0	0	0	1	0
$(\pi(a) \times \pi(b)) * f_d(a,b) = \pi(d)$	0	0.019	0.981	0	

And as results we find

$$
\begin{array}{lclllcl}
\pi(g)[\emptyset^g] & = & 0.0 & & \pi(h)[\emptyset^h] & = & 0.0 \\
\pi(g)[\{g\}] & = & 0.215 & & \pi(h)[\{h\}] & = & 0.985 \\
\pi(g)[\{\neg g\}] & = & 0.785 & & \pi(h)[\{\neg h\}] & = & 0.015 \\
\pi(g)[\Omega^g] & = & 0.0 & & \pi(h)[\Omega^h] & = & 0.0
\end{array}
$$

The transposes of the matrices can be determined in the same way as for probability tables. Of course the method should be adapted for sparse matrices.

□

7 Conclusion

In this paper, we presented a PN-representation for the propagation of probabilities and (probability) masses. PNs are presumably the most suitable modeling means for discrete concurrent processes. Moreover, PNs are basicly linear, and in so far particularly suited for the linear calculation of conditional probabilities and specializations of masses. By means of an example we demonstrated that the same nets (PPNs) are suited for the propagation of probabilities and masses if the inscriptions and matrices of the nets are properly chosen (probabilities and conditional probability tables or masses and specialization matrices, respectively). The example is specific in the "length" of probability and mass tuples and in the branching degree of the nodes and the length of the paths in the underlying dependency PN. To show the general case would be out of our focus. Neither of the specific cases is (probability) theoretically problematic in being generalized, but to present the most general case would mean to run out of space. Finally, it is absolutely obvious how to construct the PPNs for any dependency PN.

In further papers, we want to show that generalizations of our approach are possible in several respects: the propagation of intervals, to upgrade the nets to PPNs with time, and to extend the underlying propositional logic to modal logics.

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